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Superfluid density in quasi-one-dimensional boson syste

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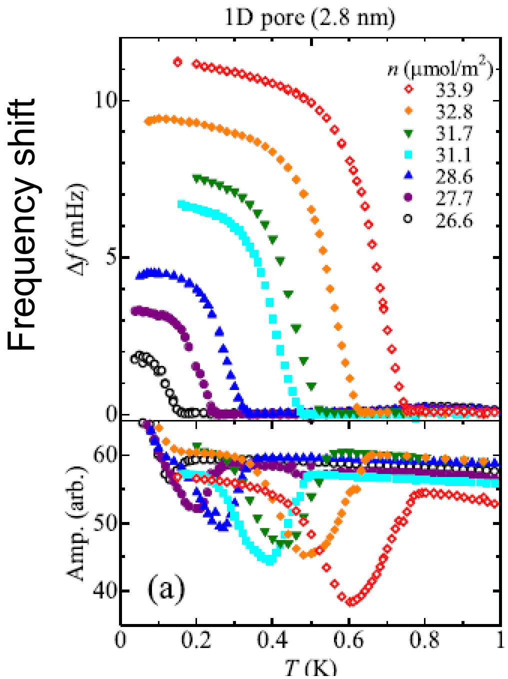
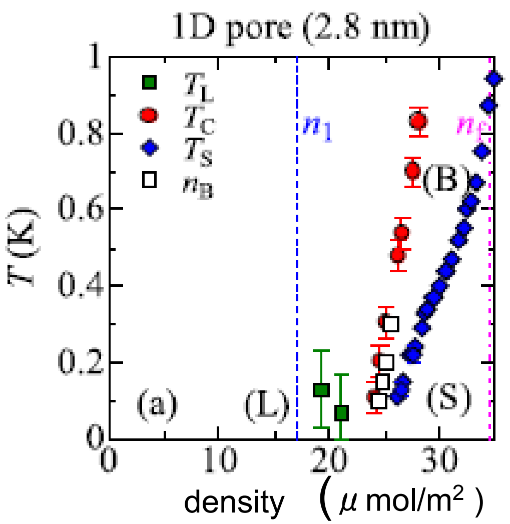
1. Superfluidity (finite superfluid density at finite T) in 1D nanopores.
2. Classical XY model in quasi-one-dimension: Helicity modulus
3. Two definitions of superfluid density
4. Summary

1. Superfluidity (finite superfluid density ρ_s at finite temperatures) in 1D nanopores.

$L \approx 300$ nm, $a \cong 2.8$ nm He film





T_s

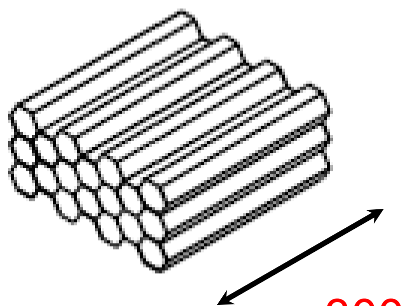
M. Suzuki: He filling nanopores

N. Wada : **He films** adsorbed on nanopo

H. Ikegami et al., Phys. Rev. B 76, 144503 (2007).
R. Toda et al. Phys. Rev. Lett. 99, 255301 (2007).

1. Signal coming from the 2D part (outer surface) ??

   No



~ 300 nm

2. Finite size effect ??

   No, because the onset temperature T_s is close to the Kosterlitz-Thouless transition temperature T_{KT}.

$$k_B T_{KT} = \frac{\pi}{2}\left(\frac{\hbar}{m}\right)^2 \rho_S(T_{KT}) \approx \frac{\pi}{2}\left(\frac{\hbar}{m}\right)^2 \rho \approx k_B T_S$$

Question: Whether or why it is possible to observe superfluidity at such high temperatures as T_{KT} in (quasi-) one dimensional systems

Minoguchi & Nagaoka (1988),
Machta & Guyer (1989)
Prokof’ev & Svistunov (2001)

Superfluid density (helicity modulus) is destroyed by free vortices at T_{KT} in 2D,

It is destroyed by “phase slippage” in quasi-1D (before it is so by vortices).

If phase slippage is not probed in experiments, one can observe superfluid behavior (at high temperatures).

2. Classical XY model in quasi-one-dimension: Helicity modulus

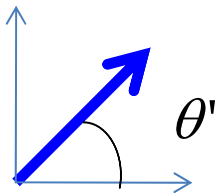
Hard core boson system ==========> XY model (s=1/2) $H = -\tilde{J} \sum_{<ij>} \vec{S}_i \cdot \vec{S}_j$

Mastubara & Mastuda, 1956

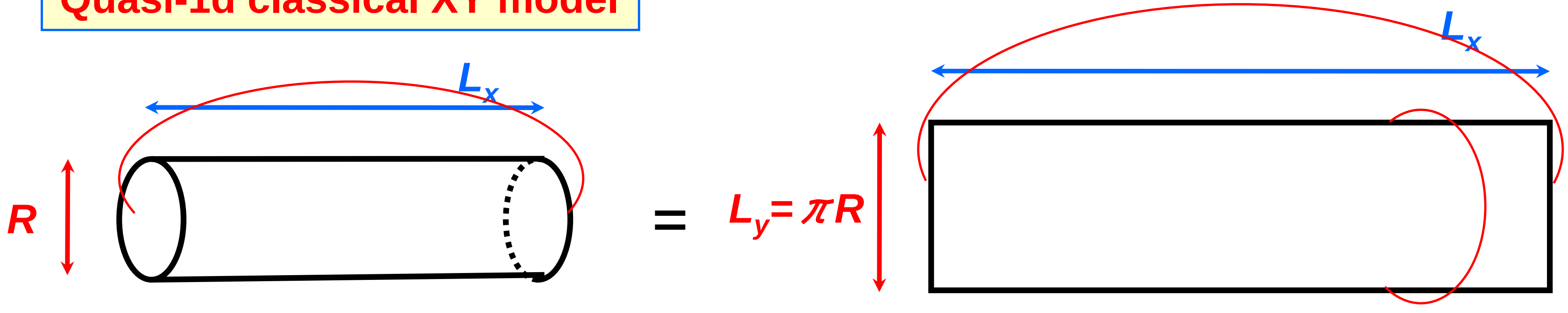
$$\vec{S} = (S_x, S_y), \quad \tilde{J} = \frac{\hbar^2}{2md^2}$$

Classical XY model

$$H_{\text{classical}} = -J \sum_{<ij>} \cos[\theta'(i) - \theta'(j)]$$



Quasi-1d classical XY model



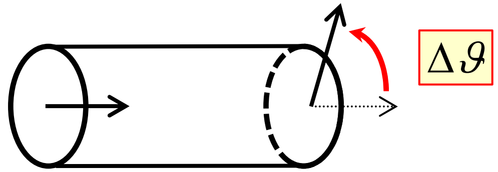
periodic b. c.

$$L_x \approx 300 \text{ nm}$$
$$R \cong 2 \text{ nm}$$

Aspect ratio A :

$$A = \frac{L_x}{L_y} = \frac{L_x}{\pi R} \approx 50 >> 1$$

Helicity modulus Υ



Phase twist $\Delta\,\theta$ between both ends → Increase in free energy per unit area

$$\Delta F = \frac{1}{2}\Upsilon\left(\frac{\Delta\vartheta}{L}\right)^2$$

Relation with superfluid density:

$$\rho_s^{\mathrm{HM}} = \left(\frac{m}{\hbar}\right)^2 \Upsilon$$

M. E. Fisher et al., (1973)

$$\Upsilon_x / J = E_x + S_x \quad \begin{cases} E_x = \dfrac{1}{L_x L_y} < \sum_i \cos[\theta'(i+\hat{x}) - \theta'(i)] > \\ S_x = -\dfrac{J}{L_x L_y T} < [\ \sum_i \sin[\theta'(i+\hat{x}) - \theta'(i)]\ ]^2 > \end{cases}$$

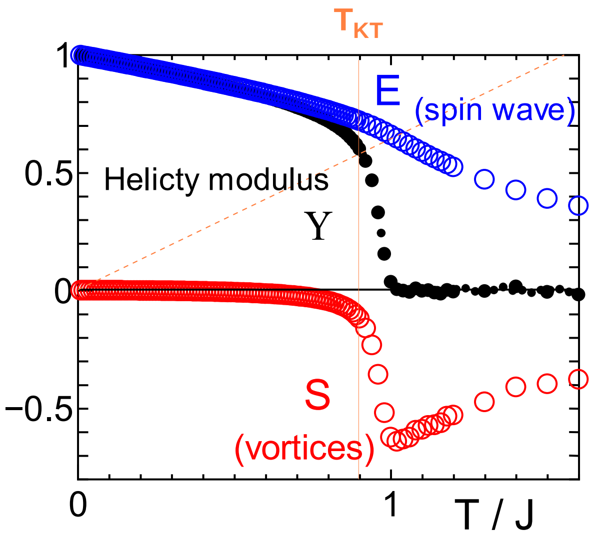
spin wave contribution

contribution from **vortices** and **phase slippage**

S. Teitel & C. Jayaprakash, (1983) .

2D case (160 x160)

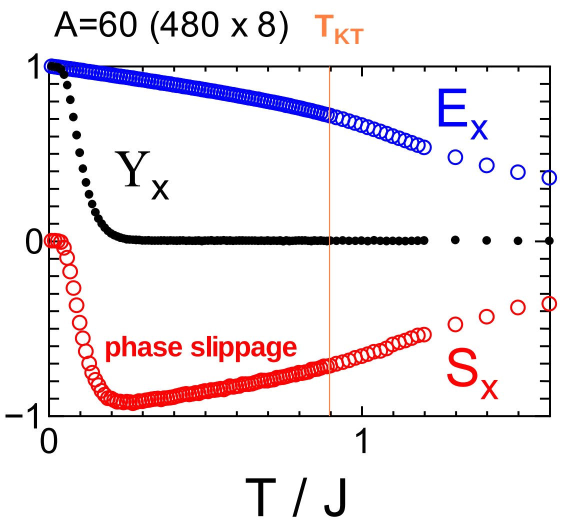
MC simulation



Free **vortices** contribute to a jump in **S**, and in **Y**.

Helicity modulus Υ in quasi-1D XY model (with A=60)

$$A = \frac{L_x}{L_y} \gg 1$$



Helicity modulus Υ rapidly vanishes at $T \le J/A$ due to the proliferating **phase slippage**.

$$Υ_x / J = E_x + S_x \quad \begin{cases} E_x = \dfrac{1}{L_x L_y} < \sum_i \cos[\theta'(i+\hat{x}) - \theta'(i)] > \\ S_x = -\dfrac{1}{L_x L_y}\dfrac{J}{T} < [\ \sum_i \sin[\theta'(i+\hat{x}) - \theta'(i)]\ ]^2 > \end{cases}$$

At **T << J**,

$$E_x = 1 - \frac{T}{4J} - \frac{1}{8}\left(\frac{T}{J}\right)^2 + \cdots$$

spin wave contribution

$$S_x = -\frac{J}{T}s(1) + [s(1) - \frac{J}{4T}s(2) + \frac{1}{4}\left(\frac{J}{T}\right)^2 s(1)^2] + \cdots \cong \begin{cases} 0 & \text{at} \quad T \ll J/A \\ -E_x + O(\exp[-\frac{A}{2}\frac{T}{J}]) & \text{at} \quad J/A \ll T \ll J \end{cases}$$

$$\theta'(i+L_x) = \theta'(i) + 2\pi n_x \qquad n_x = 0, \pm 1, \pm 2, \cdots$$

$$s(\ell) = \left\langle [\frac{(2\pi n_x)^2}{A}]^\ell \right\rangle$$

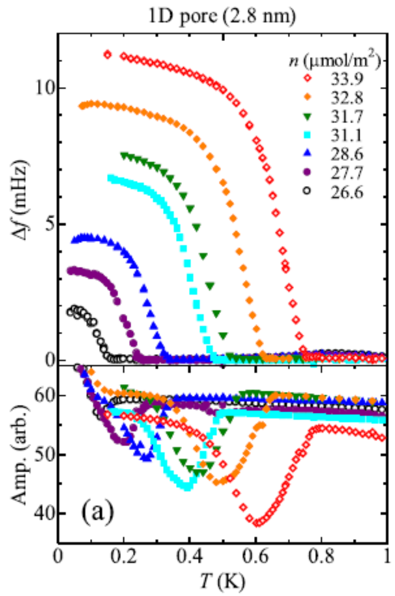
phase slippage contribution

$$\text{Helicity modulus } \Upsilon_x \cong \begin{cases} 1-\dfrac{T}{2J}+\cdots & \text{at} \quad T \ll J/A \\ O(\exp[-\dfrac{A}{2}\dfrac{T}{J}]) \ (\ll 1) & \text{at} \quad J/A \ll T \ll J \end{cases}$$

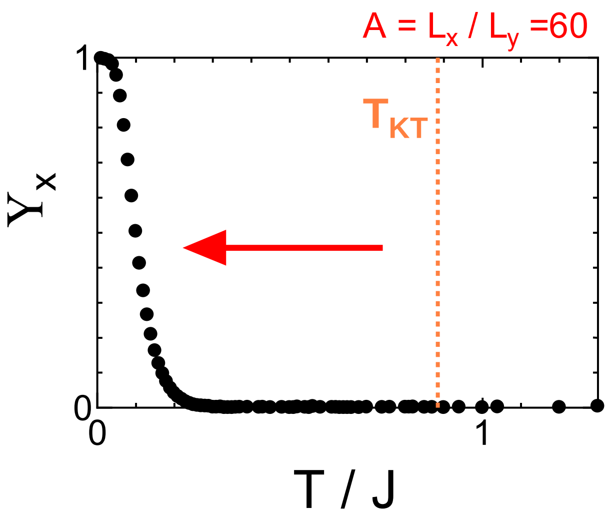
$A \to \infty$
$\to$ **pure 1D.**

Phase slippage (finite $\langle n_x^2 \rangle$) causes vanishing Υ_x at T ~J/A.

Experiment: R. Toda et al. (2007)

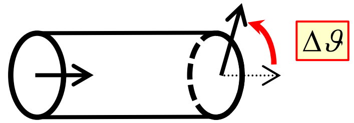


Present calculation



$T_{onset} \approx T_{KT}$ ⟷ $T_{onset} \ll T_{KT}$

3. Two (or more ?) definitions of superfluid density



(1) Helicity modulus:

$$\Delta F = \frac{1}{2}\rho_s^{HM}\left(\frac{\hbar}{m}\frac{\Delta\theta}{L}\right)^2 \Leftrightarrow \Delta F = \frac{1}{2}\Upsilon\left(\frac{\Delta\theta}{L}\right)^2$$

(2) More conventional (?)

$$\Delta F = \frac{1}{2}\rho_s v_s^2 = \frac{1}{2}\rho_s\left(\frac{\hbar}{m}\vec{\nabla}\theta\right)^2$$

Without phase slippage,

$$< n^2 > = 0 \Rightarrow \Delta\theta << 1 \Leftrightarrow \vec{\nabla}\theta << 1 \Rightarrow \rho_S^{HM} = \rho_S$$

With phase slippage,

$$< n^2 > \neq 0 \Rightarrow \Delta\theta << 1 \not\Leftrightarrow \vec{\nabla}\theta << 1 \Rightarrow \rho_S^{HM} \neq \rho_S$$

$$\rho_S^{HM} = \rho_S\left[1 - \rho_S\frac{J}{k_B T}\left\langle\frac{(2\pi n_x)^2}{A}\right\rangle\right]$$

$$\rho_S = \rho_S^{HM}(\langle n_x^2\rangle = 0) \cong \rho_S^{HM}(A=1),$$

which can be finite at $T \sim T_{KT}$.

N. V. Prokof'ev & B. V. Svistunov, 2000.

Which superfluid density is observed in (dynamical) experiments ??

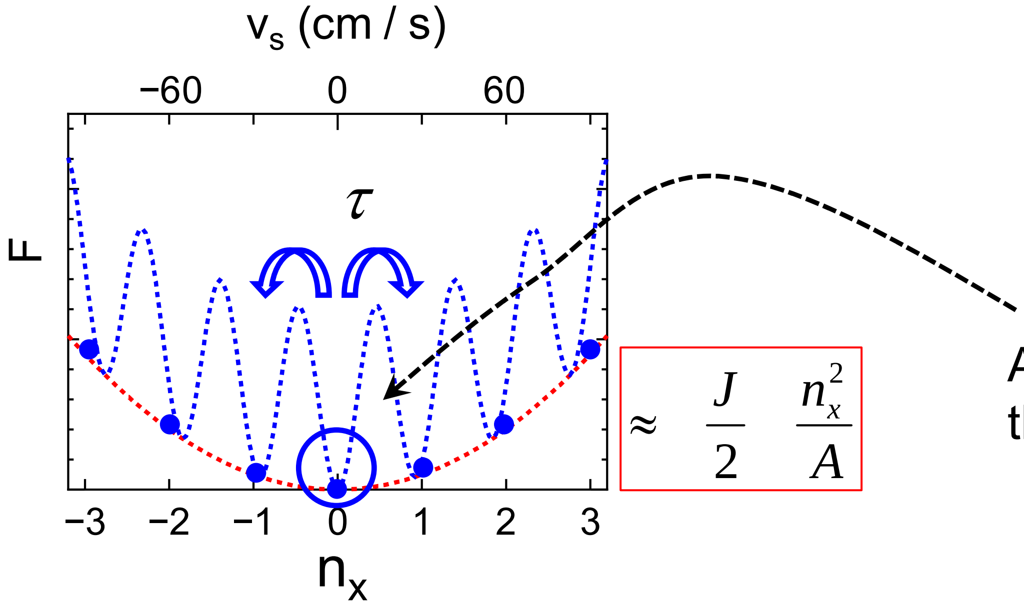
Presence of the phase slippage: **finite superflow**

$$v_s = \frac{\hbar}{m}\nabla_x \theta'(x) \approx \frac{2\pi n_x \hbar}{m L_x} \approx 30\, n_x \ \mathrm{cm}/\mathrm{s}$$

$L_x \approx 300$ nm

$$v_{\text{torsional oscill.}} \approx 10^{-4}\ \mathrm{cm}/\mathrm{s} << v_{\mathrm{S}}$$

Freq. of torsional oscillator $\omega \cong 10^3\ \mathrm{s}^{-1}$



A torsional oscillator primarily probes the region around n_x=0.

$\omega\tau << 1$ Torsional oscillator probes other minima: Effect of phase slippage.

$\rho_{\mathrm{S}}^{\mathrm{HM}}$ will be measured, and $T_{\mathrm{onset}} << T_{\mathrm{KT}}$

$\omega\tau >> 1$ Only the region around n_x=0 is probed: No effect of phase slippage

ρ_{S} will be measured, and $T_{\mathrm{onset}} \approx T_{\mathrm{KT}}$

J. Machta & R. A. Guyer, 1989

3. SUMMARY

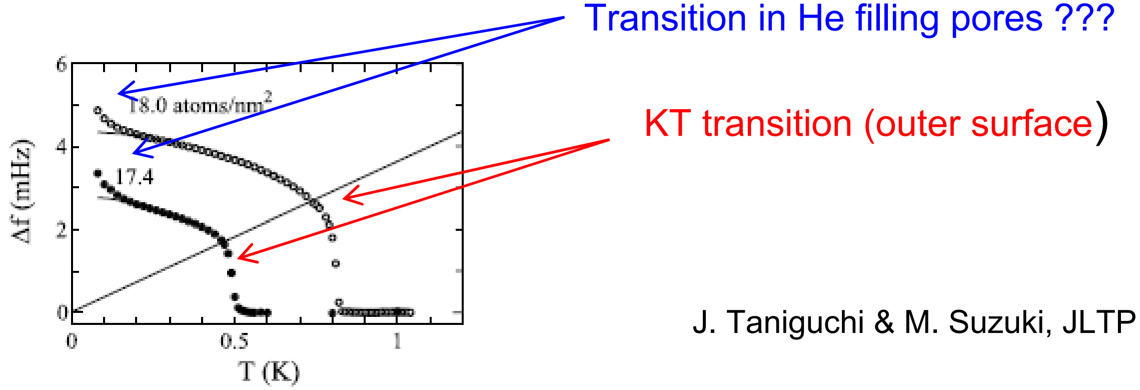
Question: Whether or why it is possible to observe superfluidity
at such high temperatures as T_{KT}
in (quasi-) one dimensional systems

Answer: It is possible when only the states without phase slippage are probed,
because superfluid density is reduced by phase slippage in 1D.

Observability of 1D behavior ??

$$\omega \tau << 1$$

Nanopores filled with He



J. Taniguchi & M. Suzuki, JLTP 150 (2008) 347.