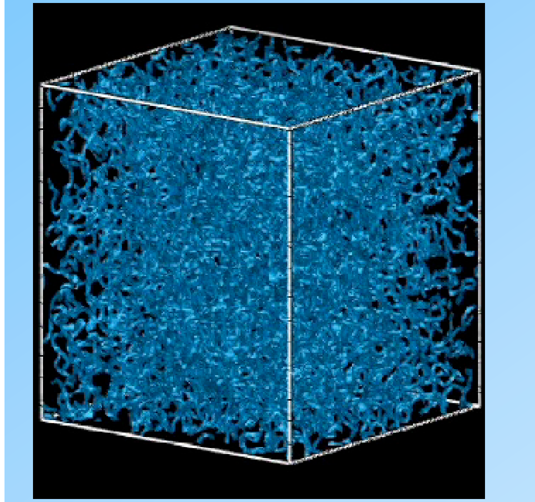
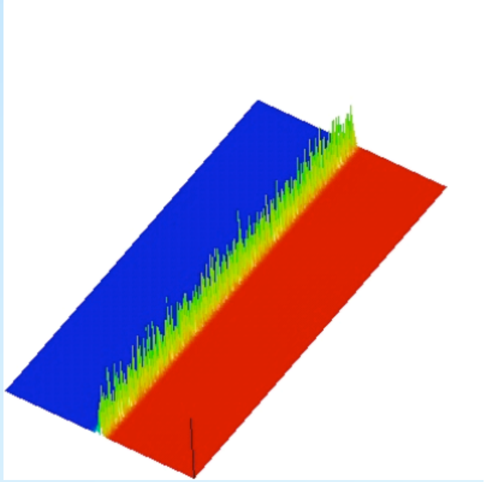
Quantum Turbulence and Nonlinear Phenomena in Quantum Fluids



Makoto TSUBOTA
Department of Physics,
Osaka City University, Japan



Review article: M. Tsubota, J. Phys. Soc. Jpn. 77, 111006(2008)
Progress in Low Temperature Physics, vol.16 (Elsevier, 2008), eds. W. P. Halperin and M. Tsubota

A03 Bose Superfluids and Quantized Vortices

Studies of physics of quantized vortices and “new” superfluid turbulence

M. Tsubota, T. Hata, H. Yano

Public participation: M. Machida, D. Takahashi

Superfluidity of atomic gases with internal degrees of freedom

M. Ueda, T. Hirano, H. Saito, S. Tojo, Y. Kawaguchi

Public participation: Y. Kato

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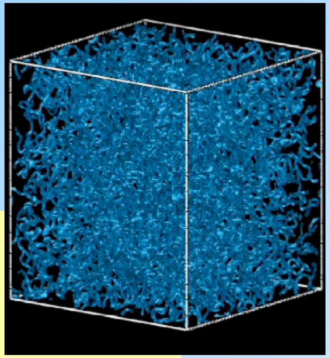
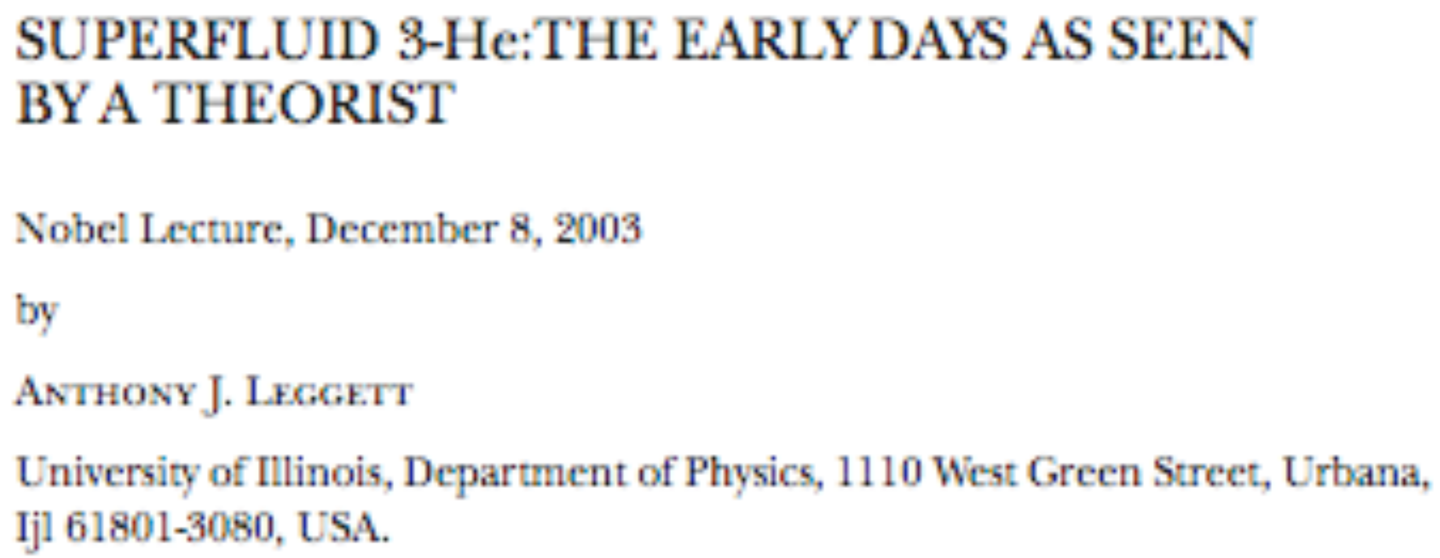


1. Why is QT (quantum turbulence) so important?
2. Outputs of our group through this five-years project
3. Very new results

3.1 Steady state of counterflow quantum turbulence: Vortex filament simulation with the full Biot-Savart law
H. Adachi, S. Fujiyama, MT, Phys. Rev. B (in press) (***Editors suggestion***)

3.2 Quantum Kelvin-Helmholtz instability in two-component Bose-Einstien condensates
H. Takeuchi, N. Suzuki, K. Kasamatsu, H. Saito, MT, Phys. Rev. B (in press)

1. Why is QT so important ?



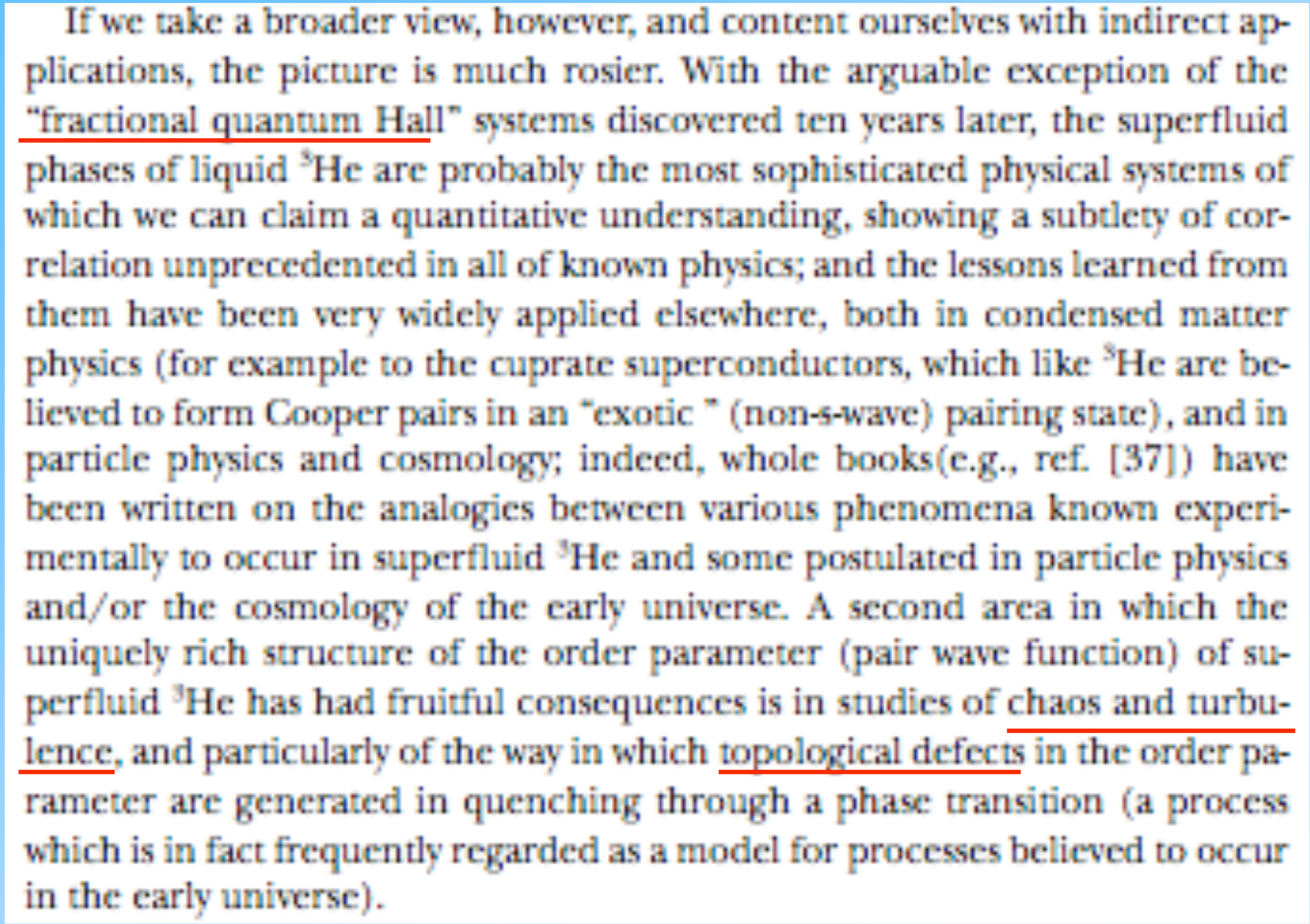
SUPERFLUID 3-He:THE EARLY DAYS AS SEEN BY A THEORIST

Nobel Lecture, December 8, 2003

by

ANTHONY J. LEGGETT

University of Illinois, Department of Physics, 1110 West Green Street, Urbana,
Ijl 61801-3080, USA.



If we take a broader view, however, and content ourselves with indirect applications, the picture is much rosier. With the arguable exception of the "fractional quantum Hall" systems discovered ten years later, the superfluid phases of liquid ^{3}He are probably the most sophisticated physical systems of which we can claim a quantitative understanding, showing a subtlety of correlation unprecedented in all of known physics; and the lessons learned from them have been very widely applied elsewhere, both in condensed matter physics (for example to the cuprate superconductors, which like ^{3}He are believed to form Cooper pairs in an "exotic " (non-s-wave) pairing state), and in particle physics and cosmology; indeed, whole books(e.g., ref. [37]) have been written on the analogies between various phenomena known experimentally to occur in superfluid ^{3}He and some postulated in particle physics and/or the cosmology of the early universe. A second area in which the uniquely rich structure of the order parameter (pair wave function) of superfluid ^{3}He has had fruitful consequences is in studies of chaos and turbulence, and particularly of the way in which topological defects in the order parameter are generated in quenching through a phase transition (a process which is in fact frequently regarded as a model for processes believed to occur in the early universe).

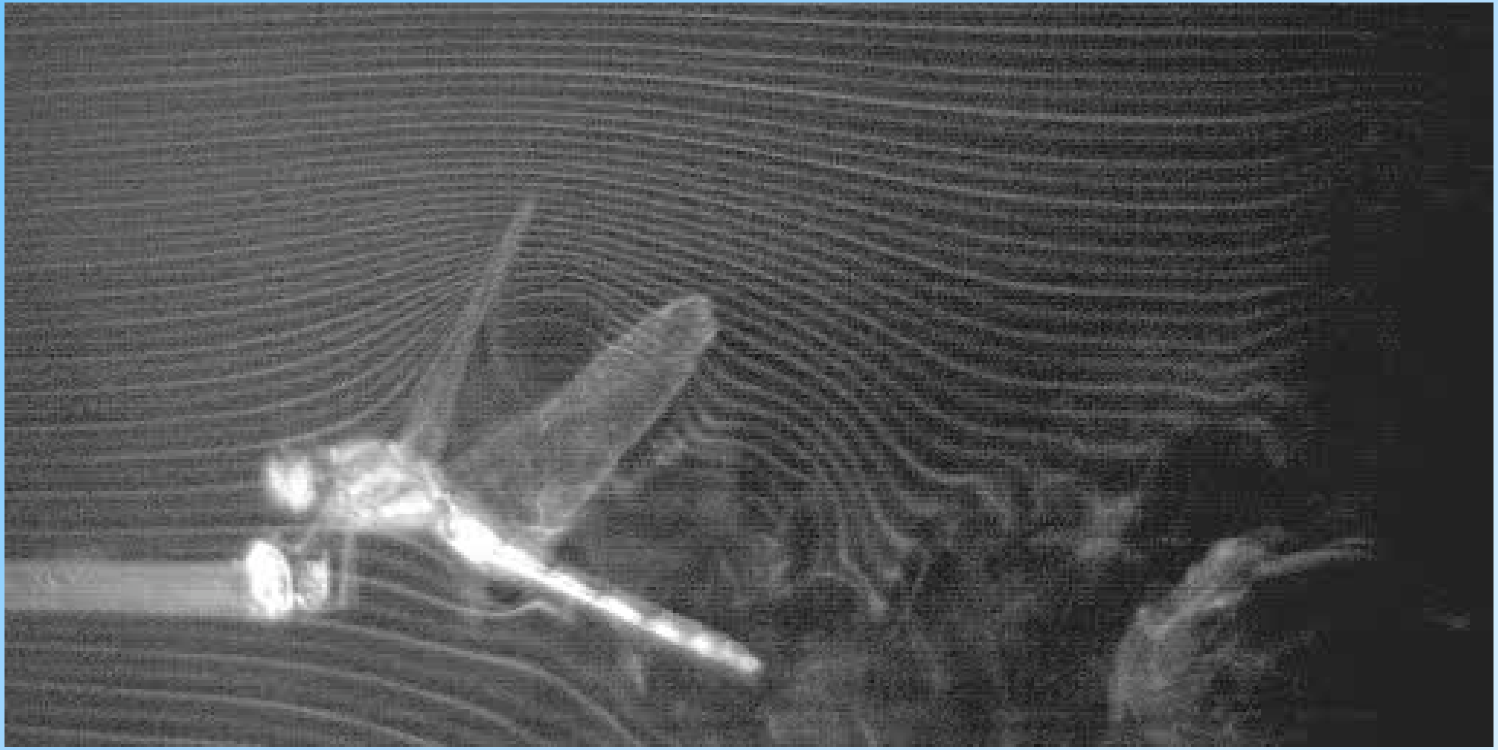
Leonardo Da Vinci
(1452-1519)

Da Vinci observed turbulent flow and found that turbulence consists of many vortices.

Turbulence is not a simple disordered state but having some structures with vortices.

Certainly turbulence looks to have many vortices.

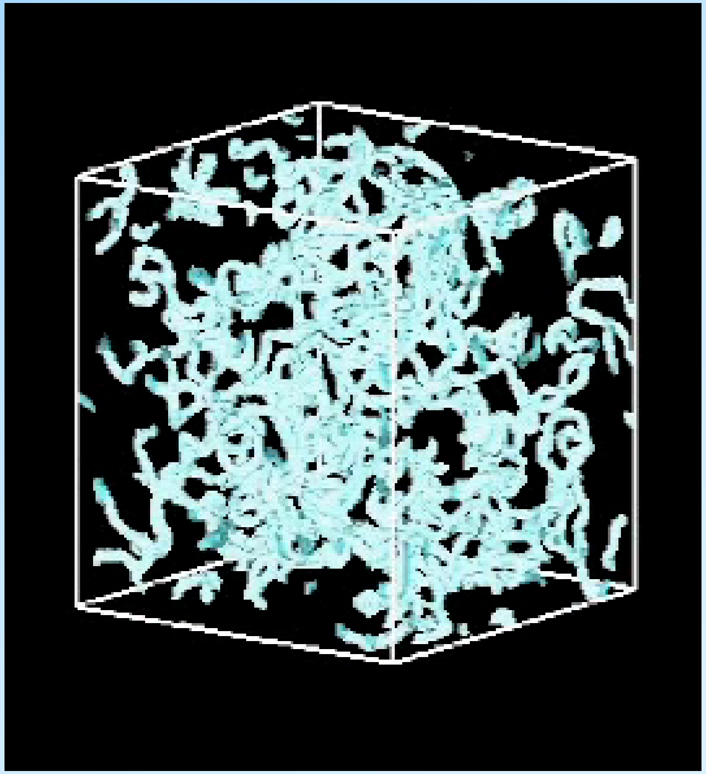
Turbulence behind a dragonfly



http://www.nagare.or.jp/mm/2004/gallery/iida/dragonfly.html

It is not so straightforward to confirm the Da Vinci message in classical turbulence.

The Da Vinci message is actually realized in quantum turbulence comprised of quantized vortices.



Quantum turbulence

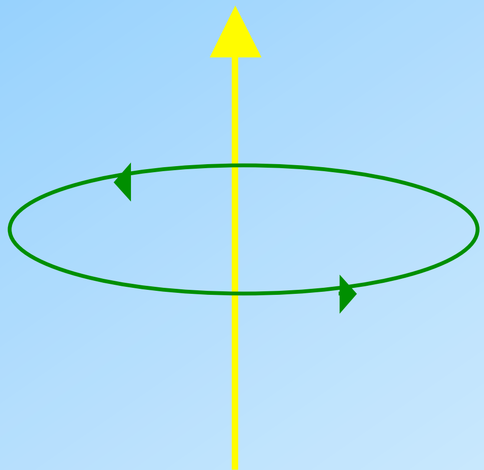
A quantized vortex is a vortex of superflow in a BEC. Any rotational motion in superfluid is sustained by quantized vortices.

(i) The circulation is quantized.

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \kappa n \qquad (n = 0, 1, 2, \cdots)$$

$$\kappa = h / m$$

A vortex with $n \geqq 2$ is unstable.



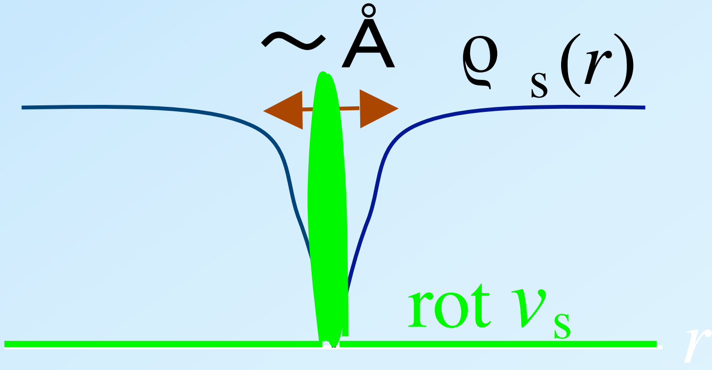
⇨ **Every vortex has the same circulation.**

(ii) Free from the decay mechanism of the viscous diffusion of the vorticity.

⇨ **The vortex is stable.**

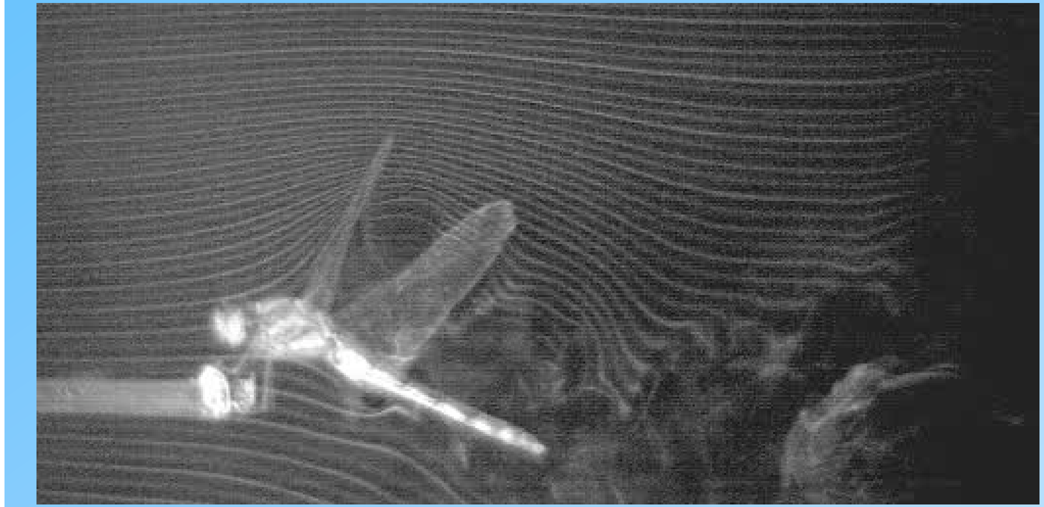
(iii) The core size is very small.

⇨ **The order of the coherence length.**

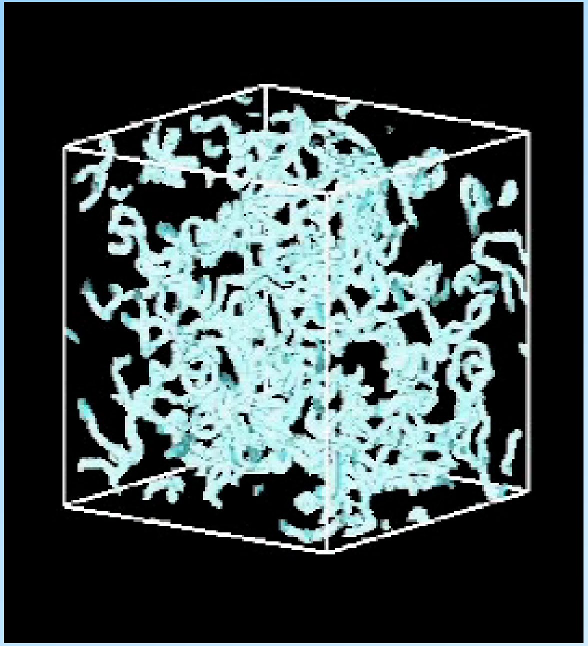


Classical Turbulence (CT) vs. Quantum Turbulence (QT)

Classical turbulence



Quantum turbulence



Motion of vortex cores

QT can be simpler than CT, because each element of turbulence is definite.

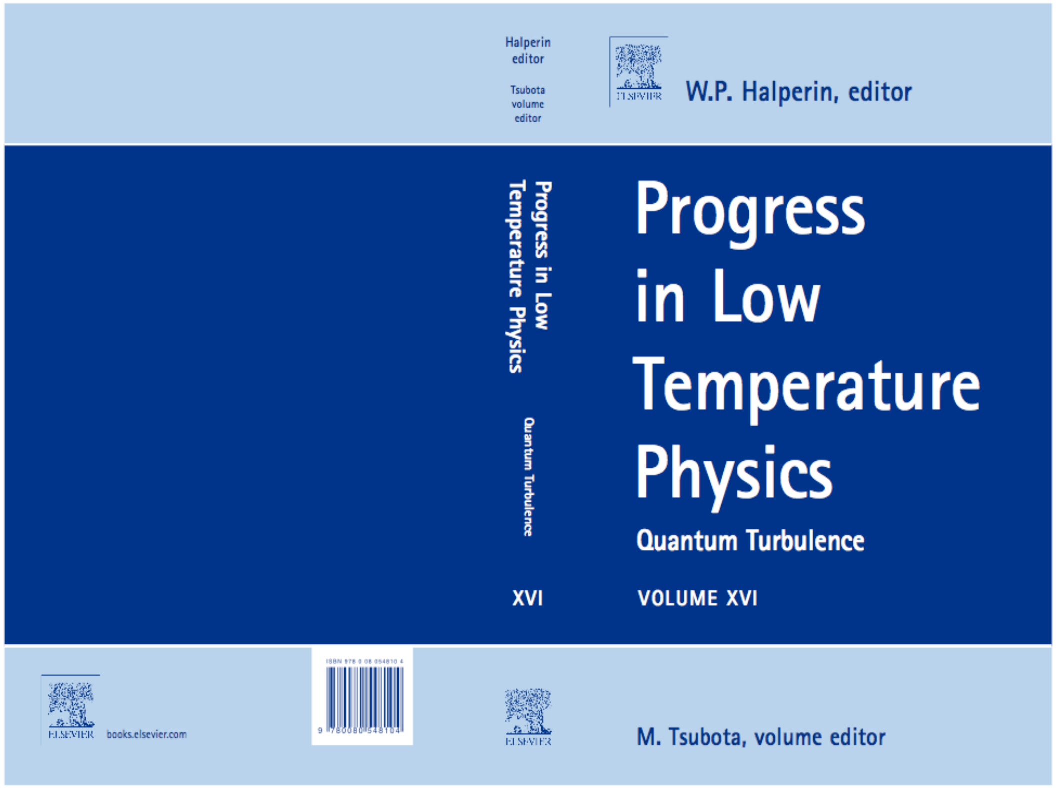
- The quantized vortices are stable topological defects.
- Every vortex has the same circulation.
- Circulation is conserved.

Quantum turbulence and quantized vortices were discovered in superfluid ^{4}He in 1950's.

This field has become a major one in low temperature physics, being now studied in superfluid ^{4}He, ^{3}He and even cold atoms.

Current important topics are well reviewed in

Progress in Low Temperature Physics, vol.16 (Elsevier, 2008), eds. W. P. Halperin and M. Tsubota



Halperin
editor
Tsubota
volume
editor

Progress in Low Temperature Physics

Quantum Turbulence

XVI

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W.P. Halperin, editor

Progress
in Low
Temperature
Physics

Quantum Turbulence

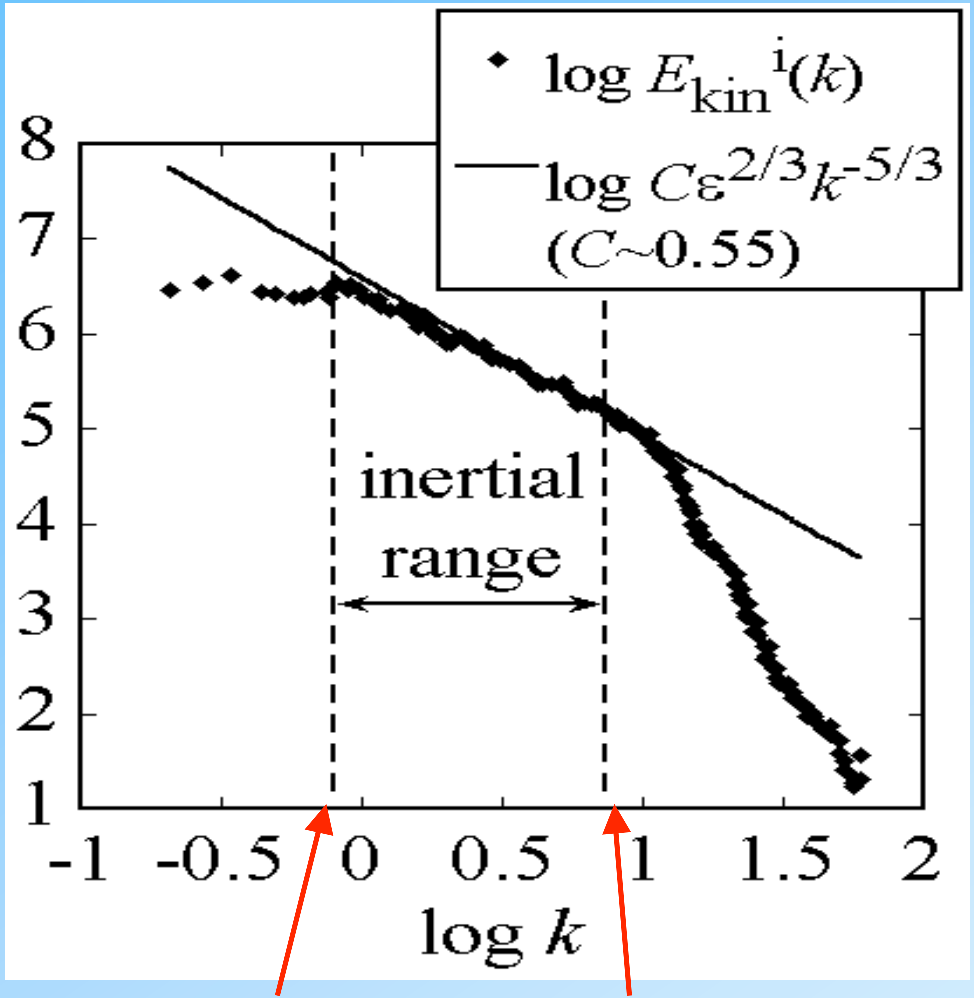
VOLUME XVI

M. Tsubota, volume editor

2. Outputs of our group through this five-years project

M. Kobayashi and MT, PRL 94, 065302 (2005), JPSJ 74, 3248 (2005)

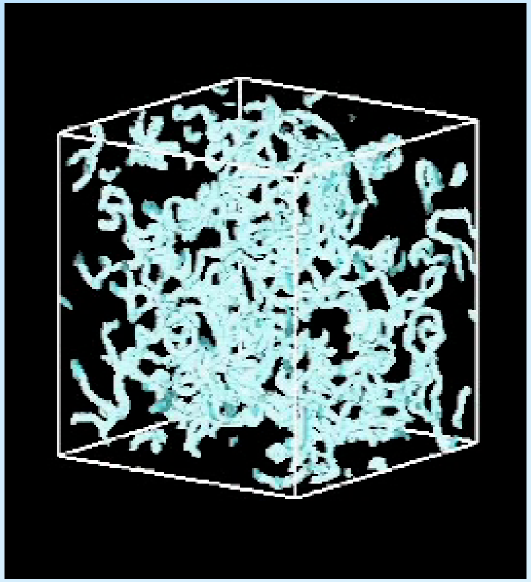
We confirmed for the first time the Kolmogorov law from the Gross-Pitaevskii model.



2π / X_0

2π / ξ

Quantum turbulence is found to express the essence of classical turbulence!



V.B.Eltsov, A.P.Finne, R.Hänninen, J.Kopu, M.Krusius, MT and E.V.Thuneberg, PRL 96, 215302 (2006)

We discovered twisted vortex state in ^{3}He-B theoretically, numerically and experimentally.

PHYSICAL
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2 JUNE 2006

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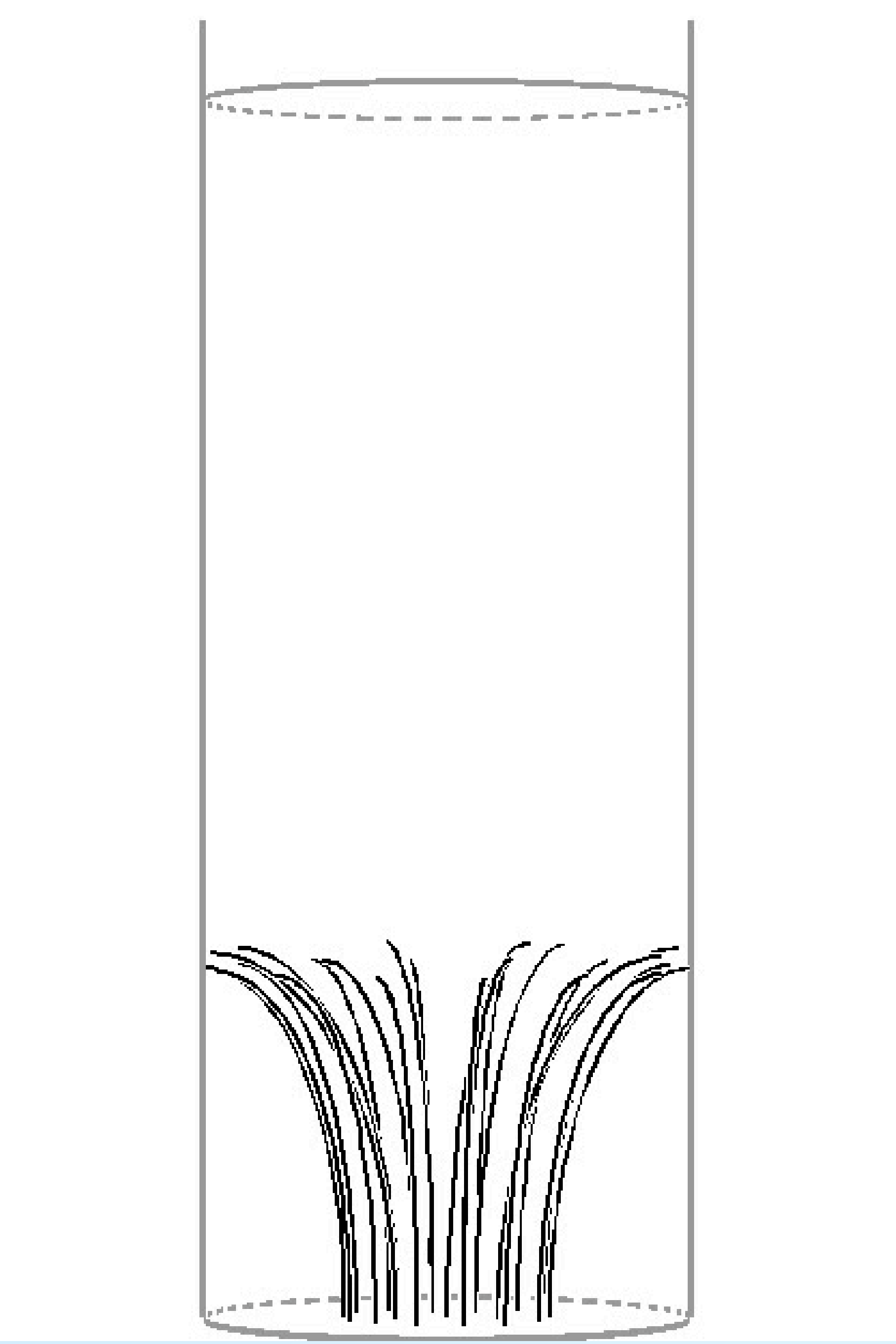
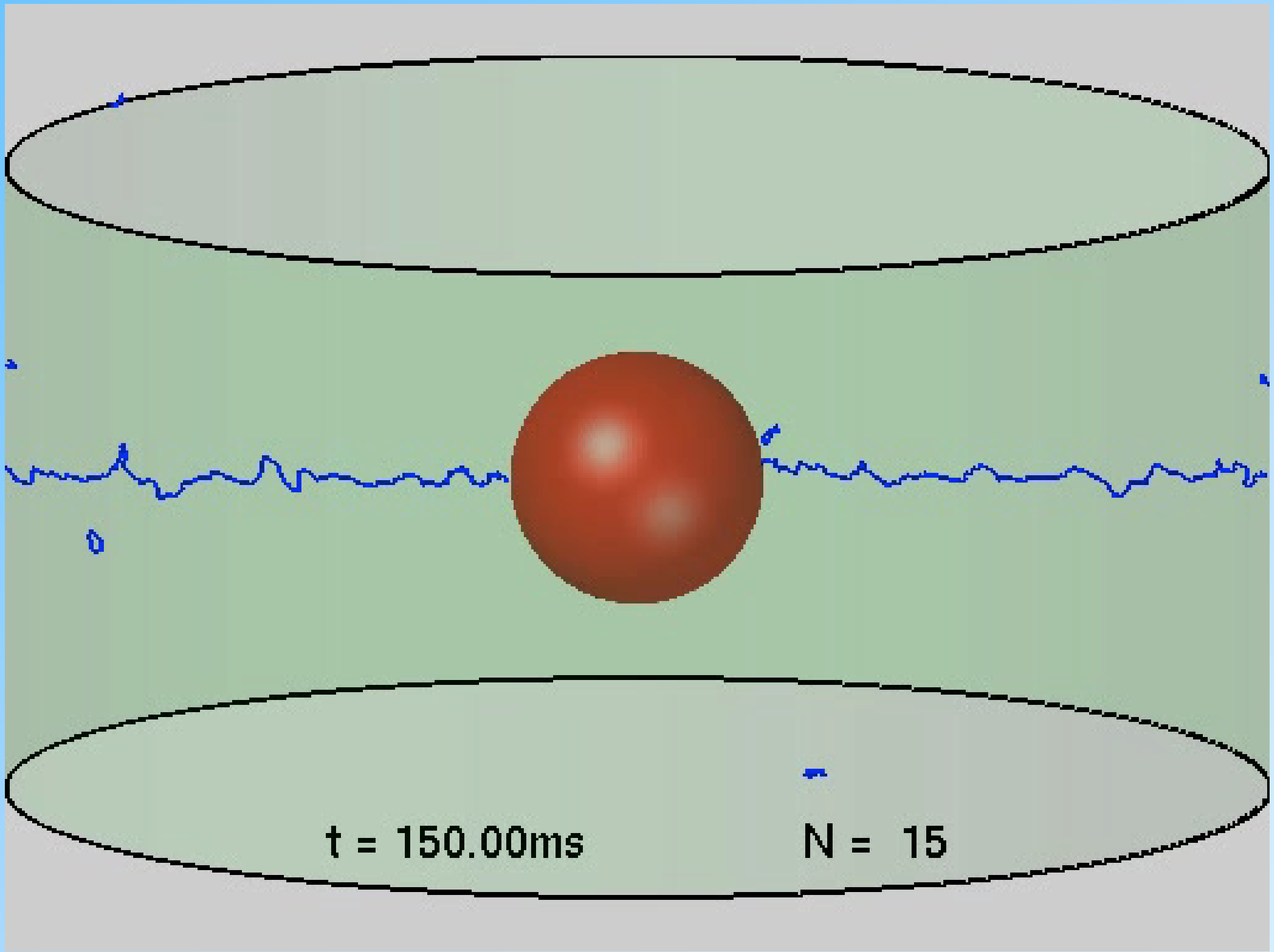
APS physics

Published by The American Physical Society



R. Hänninen, MT, W. F. Vinen, PRB **75**, 064502 (2007)

How remnant vortices develop to a tangle under AC flow



R=100 μm

ω=200 Hz

V=150 mm/s

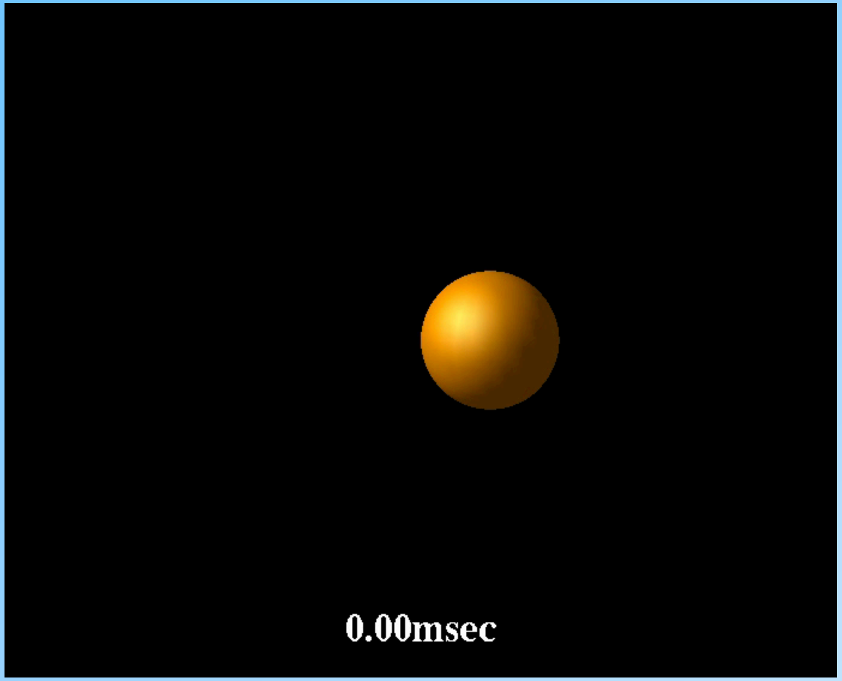
a. Kelvin waves form on the bridged vortex line.

b. Vortex rings nucleate by reconnection.

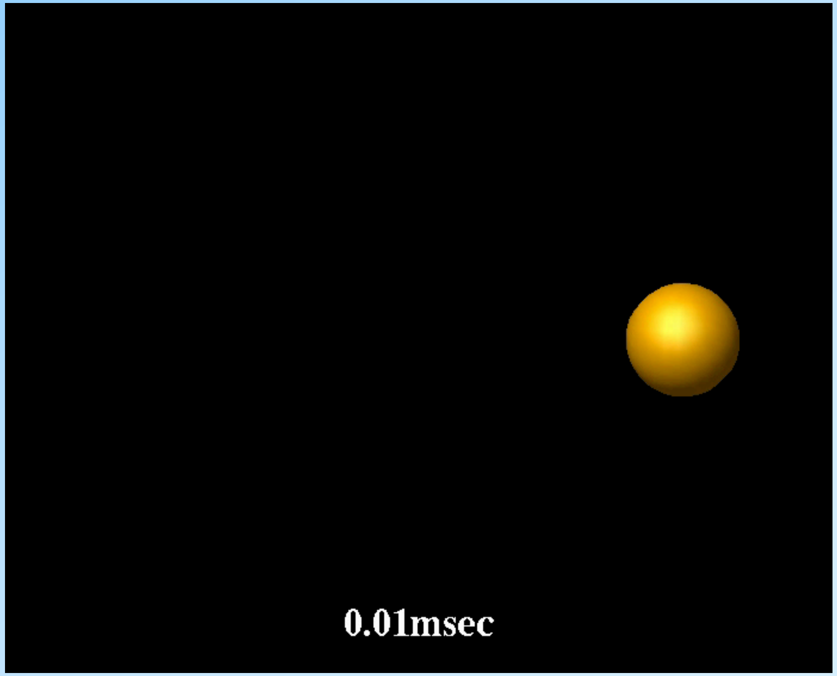
c. Turbulence develops.

R. Goto, S. Fujiyama, H. Yano, Y. Nago, N. Hashimoto, K. Obara, O. Ishikawa, MT, T. Hata, PRL 100, 045301(2008)

We found the transition to QT by seed vortex rings.



30mm/s

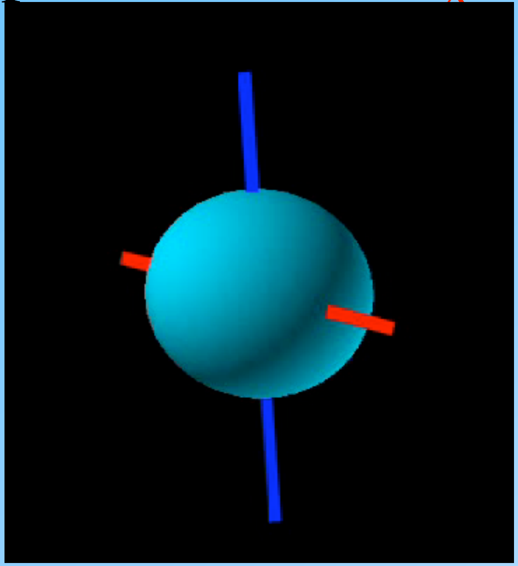


137 mm/s

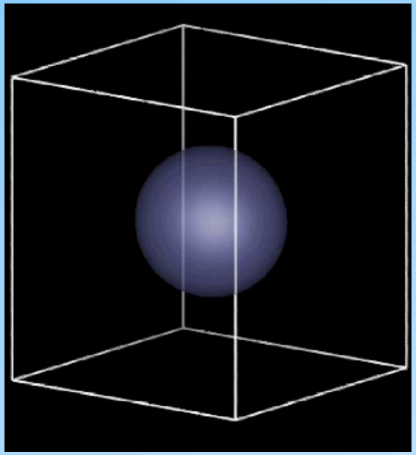
Parameters for the sphere : Radius 3μm,   Frequency 1590 Hz

M. Kobayashi and MT, PRA76, 045603(2007)

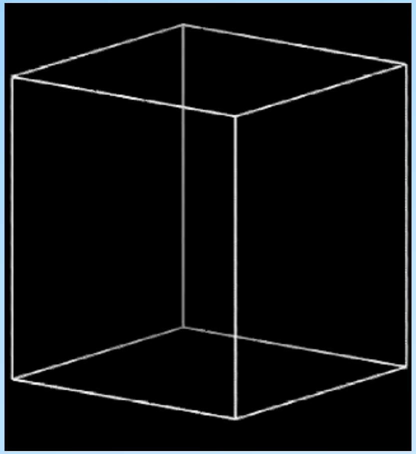
Two precessions ($\omega_x \times \omega_z$)



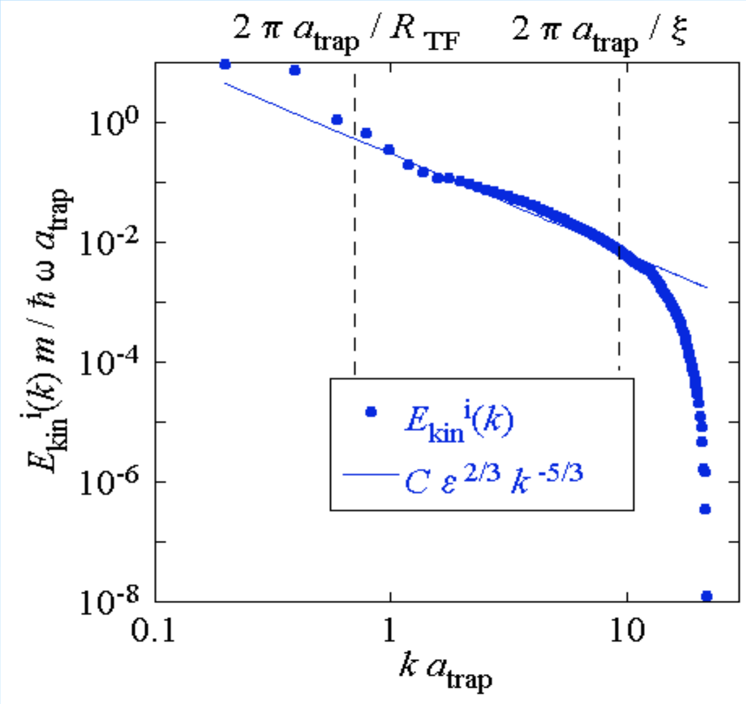
We showed how to make QT in a trapped BEC and obtained the energy spectrum consistent with the Kolmogorov law.



Condensate density



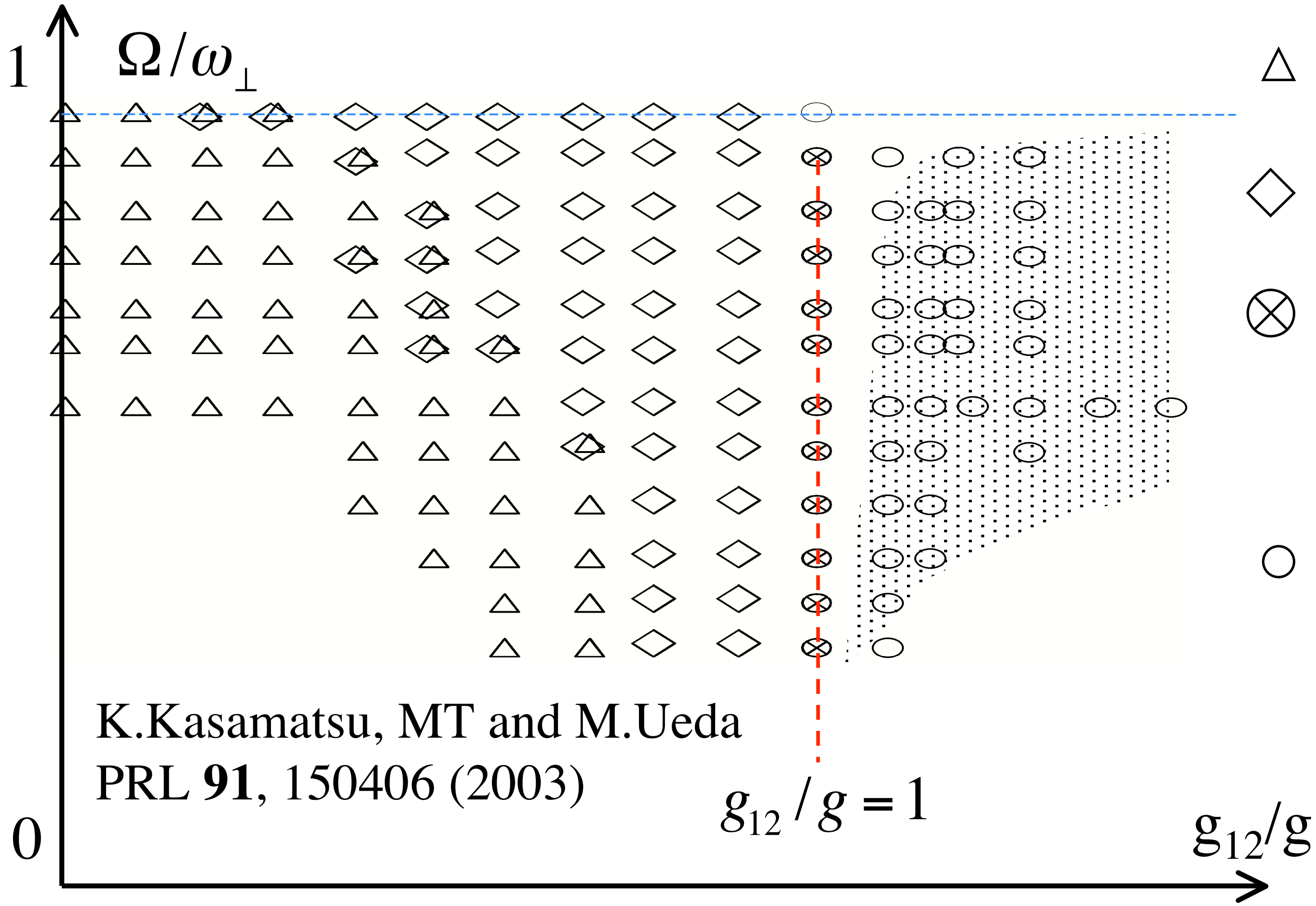
Quantized vortices



Next talk by Bagnato!

K. Kasamatsu and MT, PRA79, 023606(2009)

We revealed vortex sheet in rotating two-component BECs.



K.Kasamatsu, MT and M.Ueda
PRL **91**, 150406 (2003)

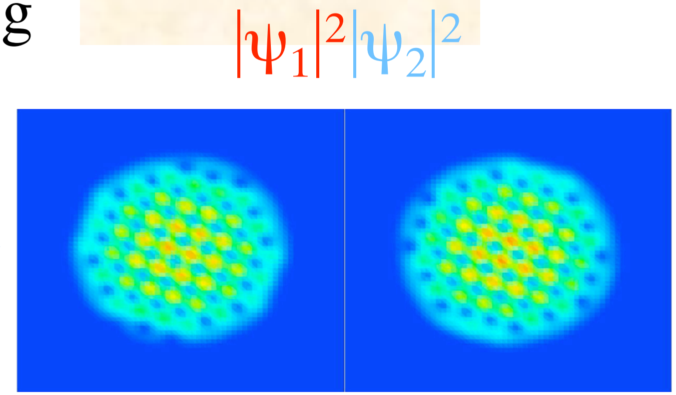
△ Triangular lattice

◇ Square lattice

⊗ Vortex stripe
or
Double-core vortex lattice

○ Vortex sheet

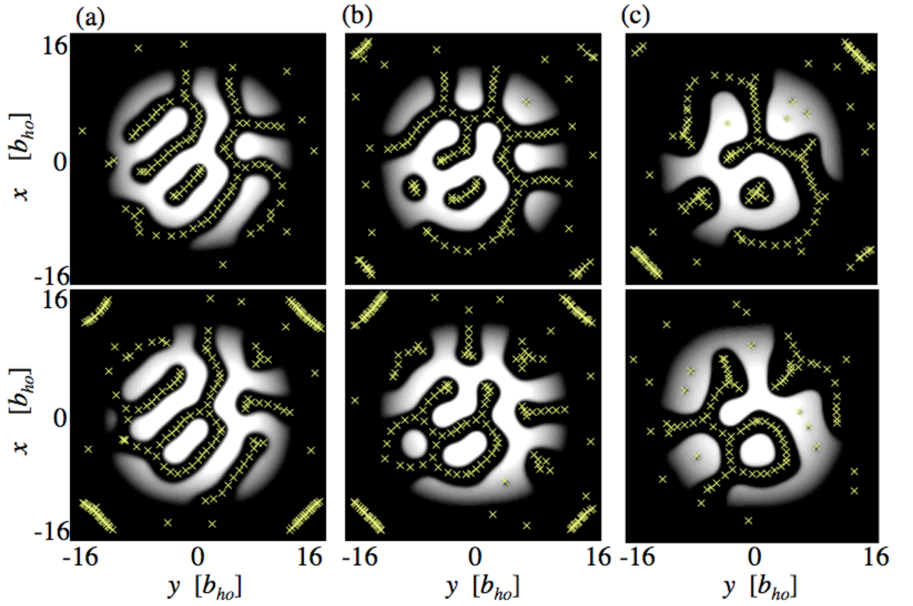
g_{12}: Interaction between two components



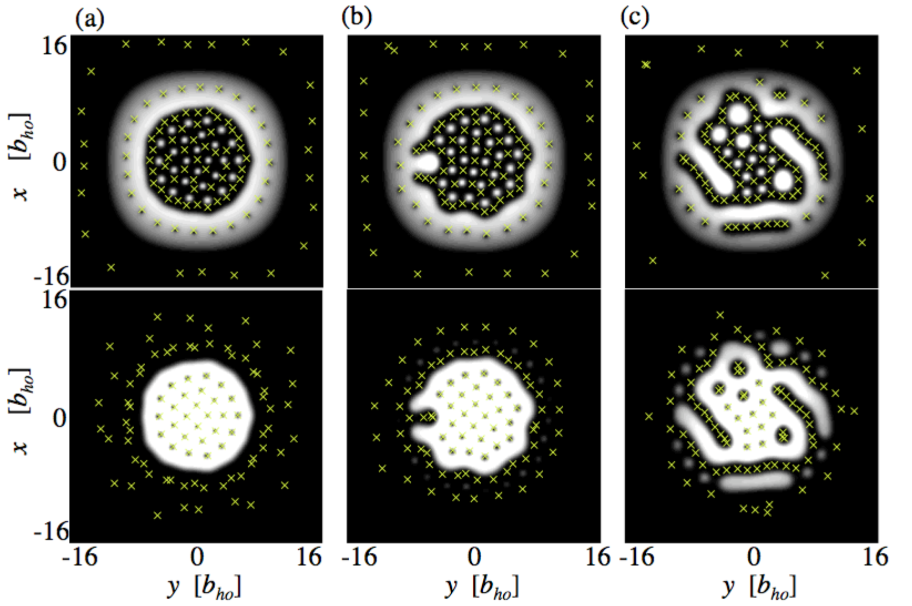
Square lattice

K. Kasamatsu and MT, PRA79, 023606(2009)

We revealed vortex sheet in rotating two-component BECs.



Density profile for g_{12}/g= 1.5 (a), 2.0 (b) and 3.0 (c).



Imbalanced case with g_{12}/g= 1.1, u_1=4000, and u_2=3000 (a), 3500 (b) and 3900 (c).

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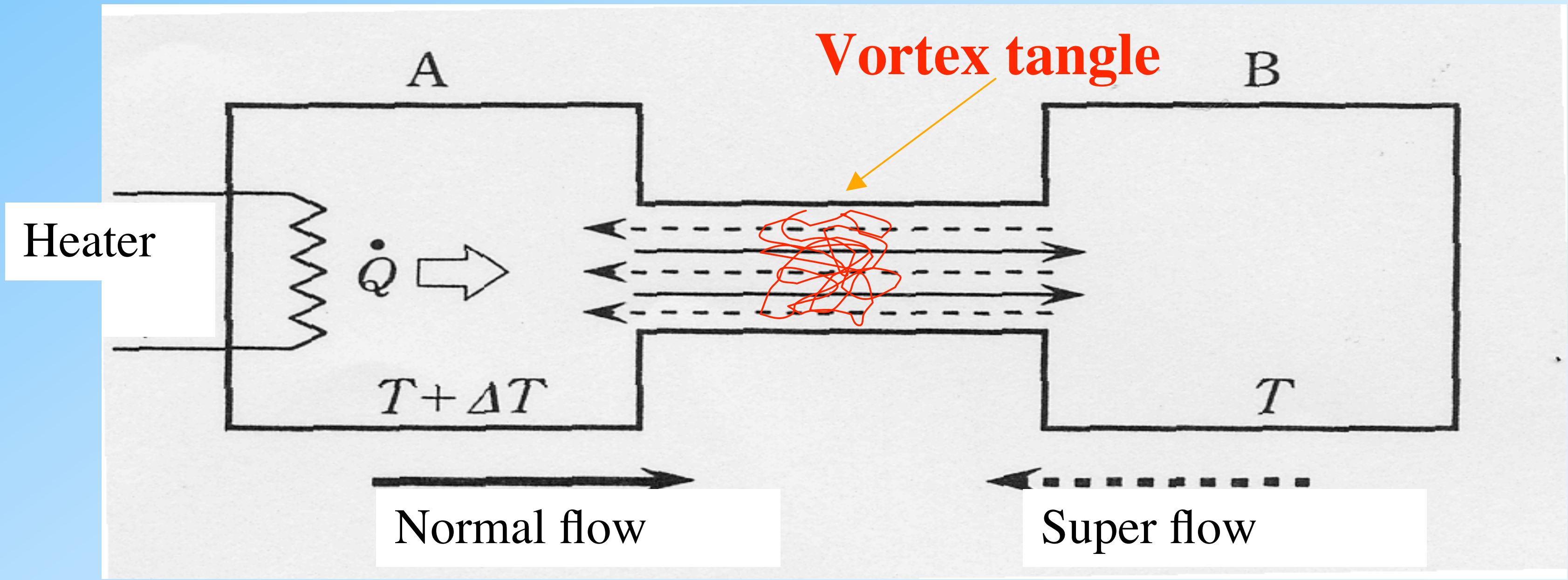
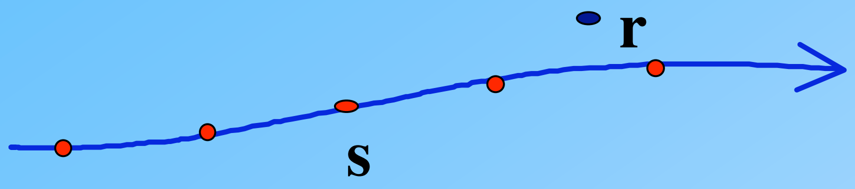
3.1 Steady state of counterflow quantum turbulence: Vortex filament simulation with the full Biot-Savart law

Hiroyuki Adachi, Shoji Fujiyama, MT, Phys. Rev. B (in press) (***Editors suggestion***) arXiv:0912.4822

Lots of experimental studies were done chiefly for thermal counterflow of superfluid ^{4}He.



Vortex filament model (Schwarz)



A vortex makes the superflow of the Biot-Savart law, and moves with this local flow. At a finite temperature, the mutual friction should be considered.

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi}\mathbf{s}'\times\mathbf{s}'' + \xcancel{\frac{\kappa}{4\pi}\int_L^{\prime}\frac{(\mathbf{s}_1-\mathbf{r})\times d\mathbf{s}_1}{|\mathbf{s}_1-\mathbf{r}|^3}} + \mathbf{v}_{s,a}(\mathbf{s})$$

$$\dot{\mathbf{s}} = \dot{\mathbf{s}}_0 + \alpha\,\mathbf{s}'\times(\mathbf{v}_n-\dot{\mathbf{s}}_0) - \alpha'\mathbf{s}'\times[\mathbf{s}'\times(\mathbf{v}_n-\dot{\mathbf{s}}_0)]$$

The approximation neglecting the nonlocal term is called the LIA(Localized Induction Approximation).

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi}\mathbf{s}'\times\mathbf{s}'' + \mathbf{v}_{s,a}(\mathbf{s})$$

Schwarz's simulation(1) PRB38, 2398(1988)

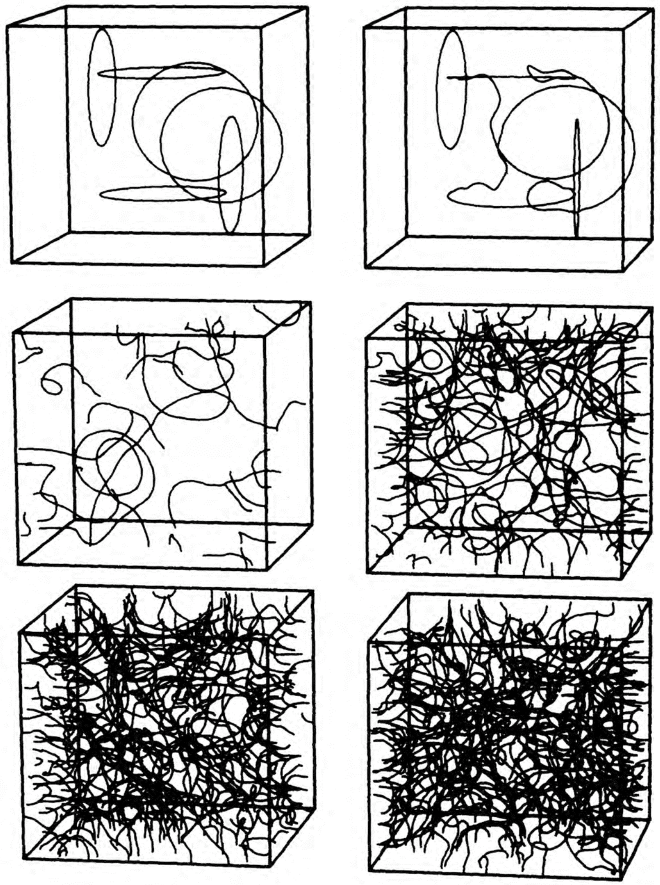


FIG. 4. Case study of the development of a vortex tangle in a real channel. Here, $\alpha = 0.10$, corresponding to a temperature of about 1.6 K, and $v_{s,0} = 75$ into the front face of the channel section shown. Upper left: $t_0 = 0$, no reconnections; upper right; $t_0 = 0.0028$, three reconnections; middle left: $t_0 = 0.05$, 18 reconnections; middle right: $t_0 = 0.20$, 844 reconnections; lower left: $t_0 = 0.55$, 12 128 reconnections; lower right: $t_0 = 2.75$, 124 781 reconnections.

Schwarz simulated the counterflow turbulence by the vortex filament model and obtained the statistically steady state.

However, this simulation was unsatisfactory.

1. All calculations were performed by the LIA.

Schwarz’s simulation(2) PRB38, 2398(1988)

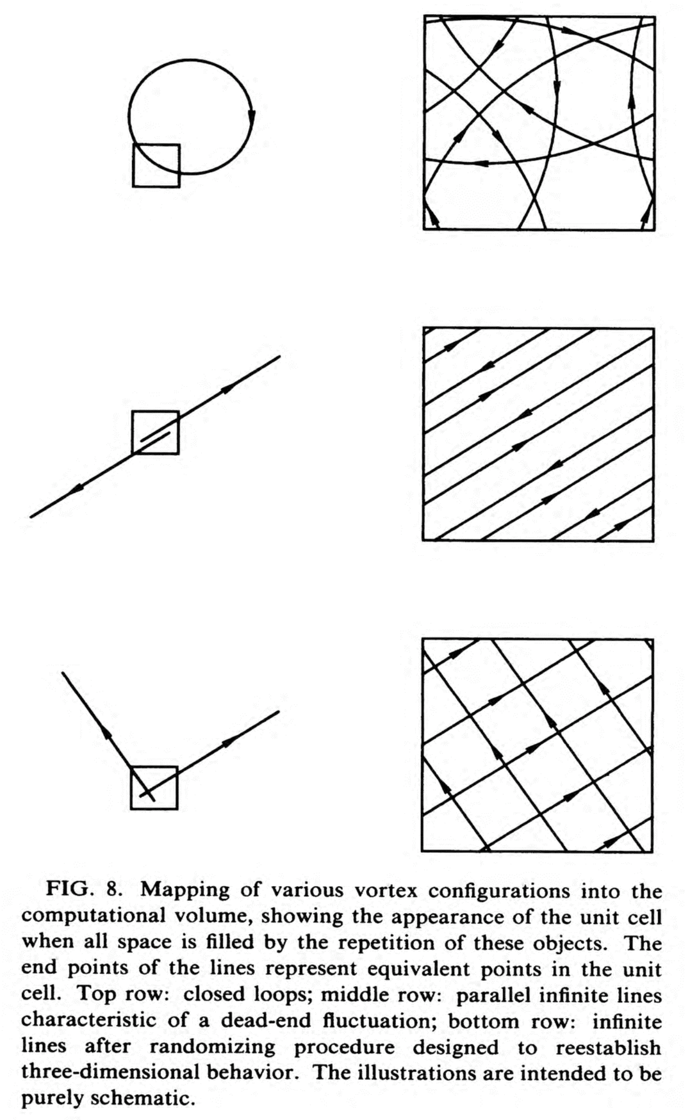


FIG. 8. Mapping of various vortex configurations into the computational volume, showing the appearance of the unit cell when all space is filled by the repetition of these objects. The end points of the lines represent equivalent points in the unit cell. Top row: closed loops; middle row: parallel infinite lines characteristic of a dead-end fluctuation; bottom row: infinite lines after randomizing procedure designed to reestablish three-dimensional behavior. The illustrations are intended to be purely schematic.

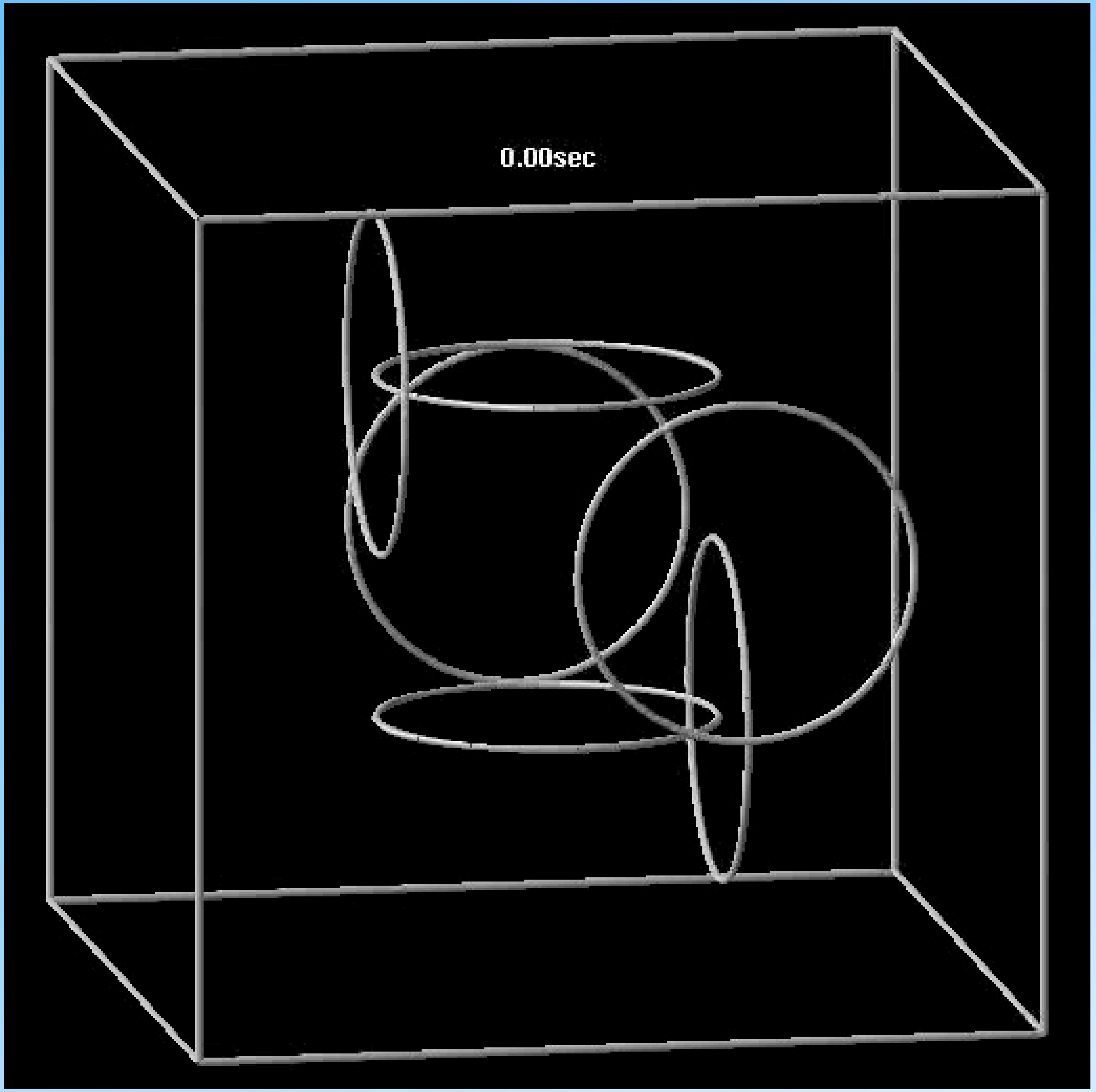
However, this simulation was unsatisfactory.

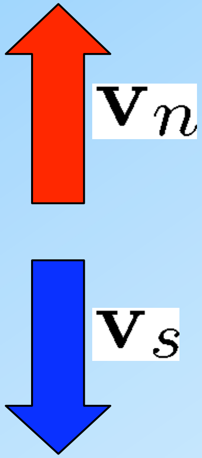
1. All calculation was performed by the LIA.
2. He used an artificial mixing procedure in order to obtain the steady state.

After Schwarz, there has been no progress on the counterflow simulation.

In this work we made the steady state of counterflow turbulence by fully nonlocal simulation.

Simulation by the full Biot-Savart law





BOX (0.1cm)^3

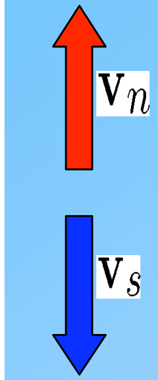
T = 1.6(K)

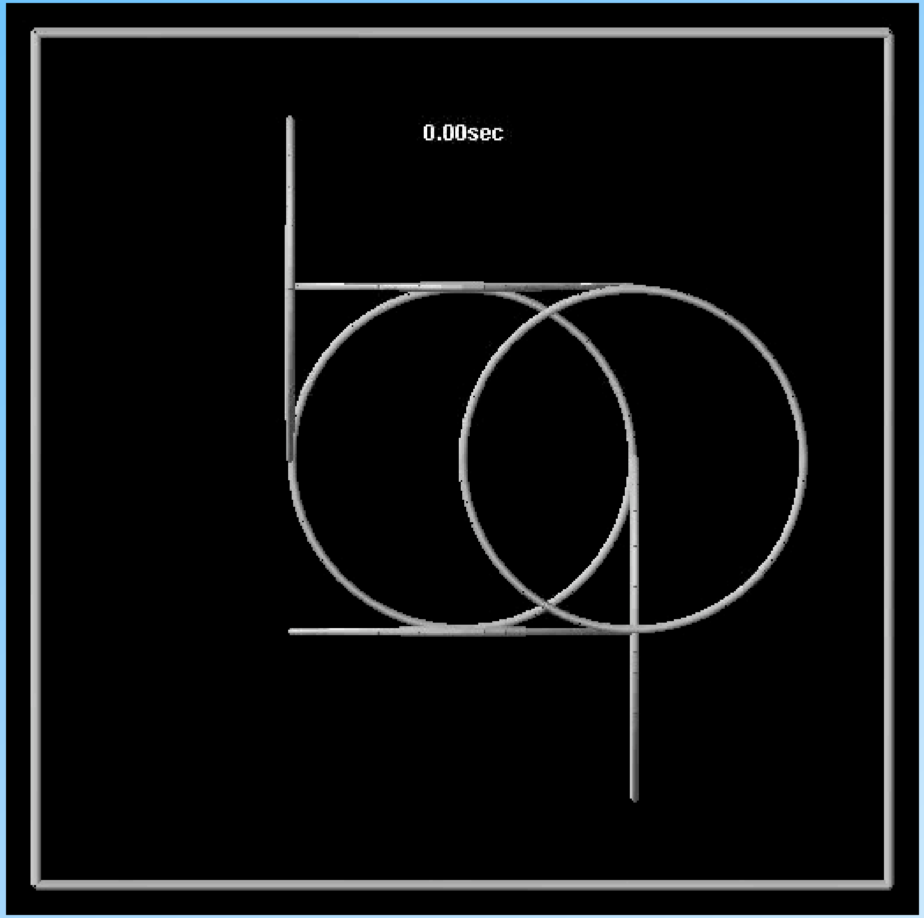
Vns = 0.367cm/s

Periodic boundary conditions for all three directions

Comparison between LIA and full Biot-Savart

Full Biot-Savart



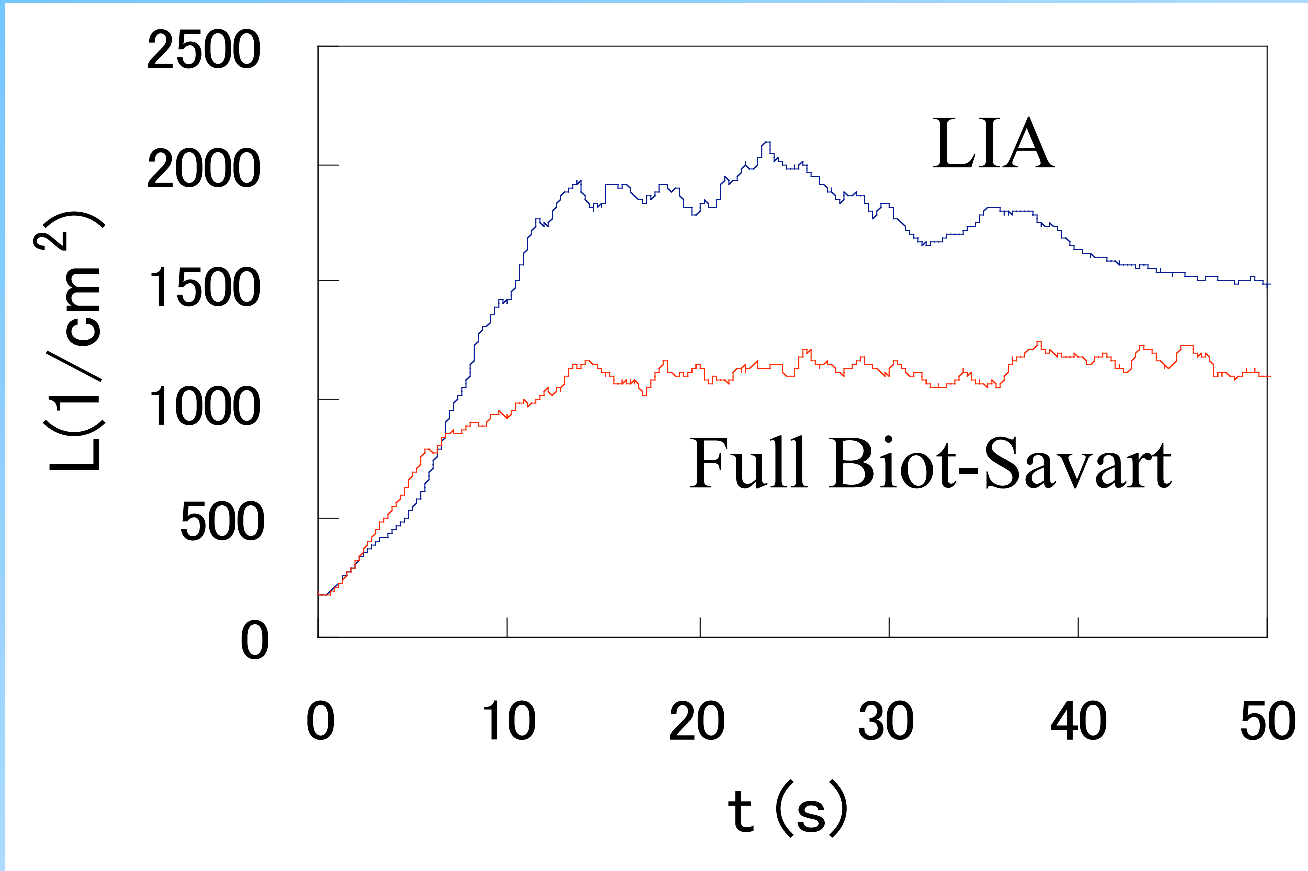


LIA

We need intervortex interaction.

Vortices become anisotropic,
forming layer structures.

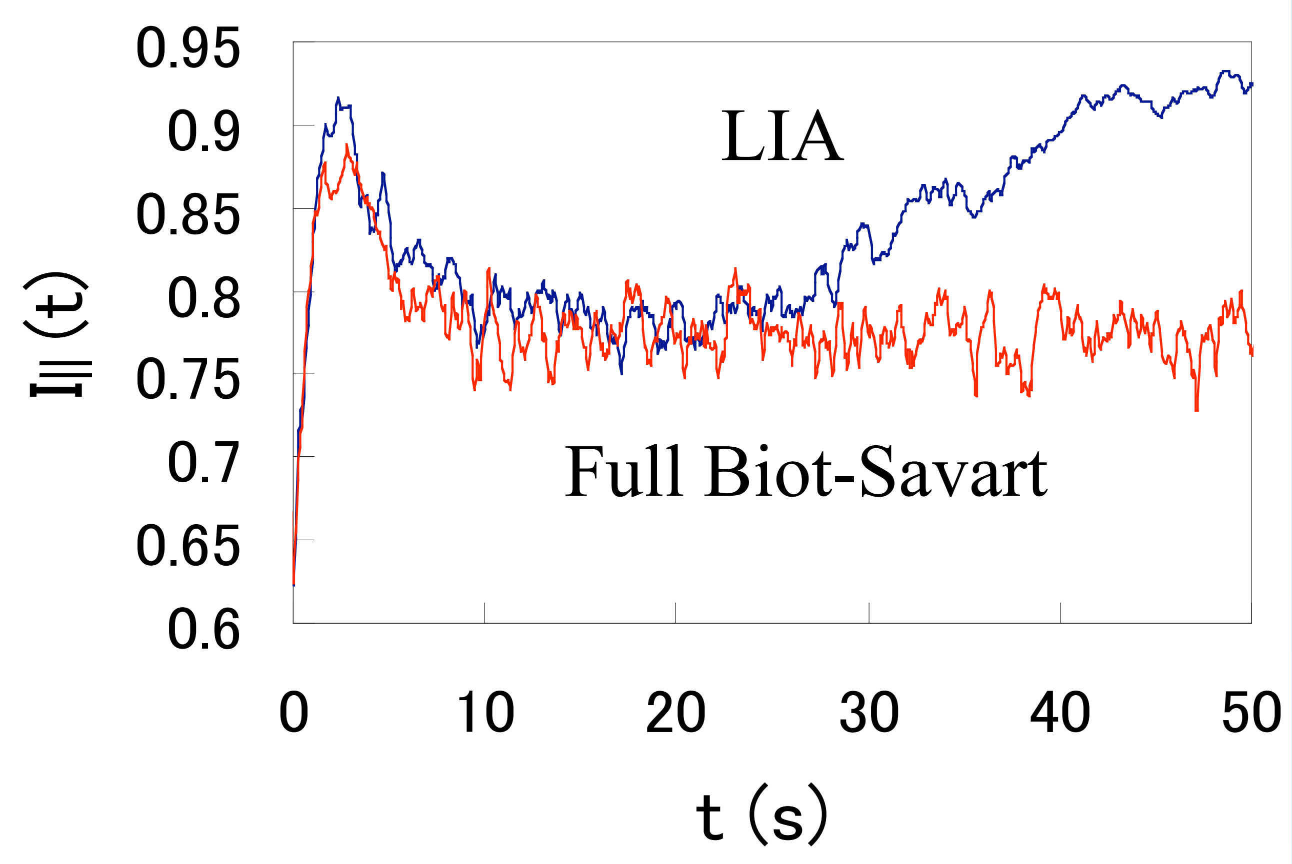
Developments of the line-length density between LIA and Full Biot-Savart



T=1.6 K、Vns=0.367 cm/s、box=(0.2 cm)^3

Anisotropic parameter

$$I_{||} = \frac{1}{\Omega L} \int_{\mathcal{L}} [1 - (\mathbf{s}' \cdot \hat{\mathbf{r}}_{||})^2] d\xi$$



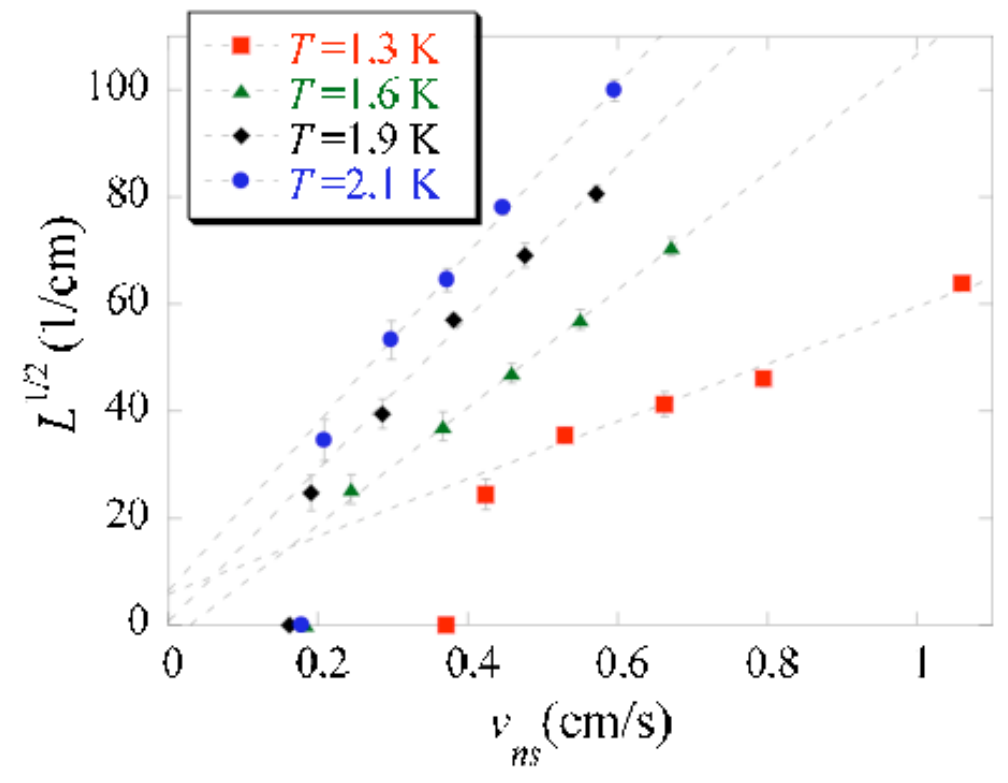
T=1.6 K、Vns=0.367 cm/s、box=(0.2 cm)^3

Quantitative comparison with observations

An important criterion of the steady state is to obtain

$$L = \gamma^2 |\mathbf{v}_{ns}|^2$$

L: Vortex density, v_{ns}:relative velocity in counterflow



	γ (s/cm^2) Our calculation	γ (s/cm^2) Experment
1.3 K	54	59
1.6 K	109	93
1.9 K	140	133
2.1 K	157	(154)

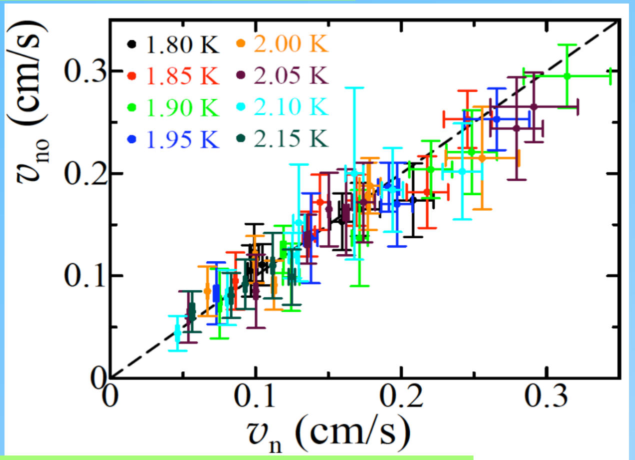
Childers and Tough, Phys. Rev. B13, 1040 (1976)

The parameter γ agrees with the experimental observation quantitatively.

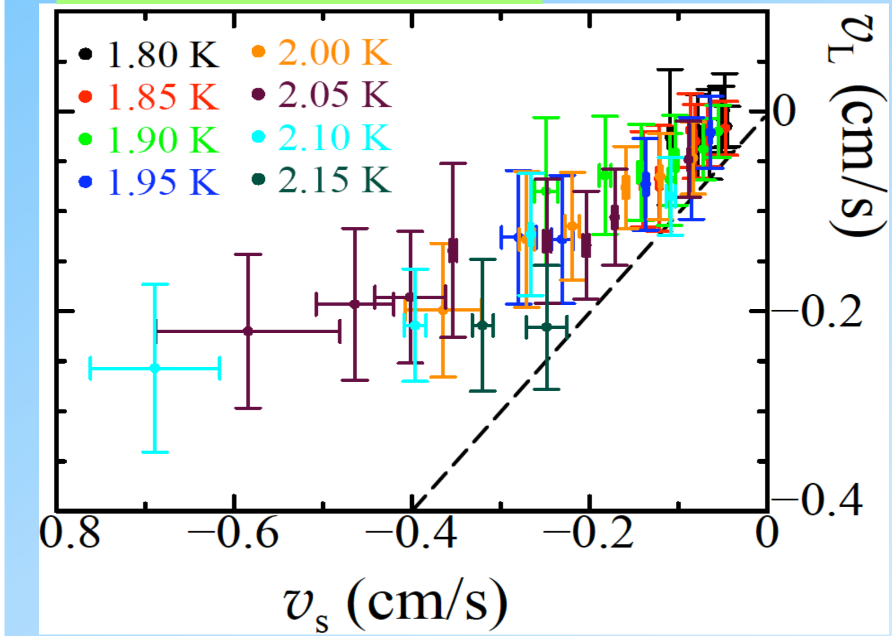
Observation of the velocity by the solid hydrogen particles in counterflow

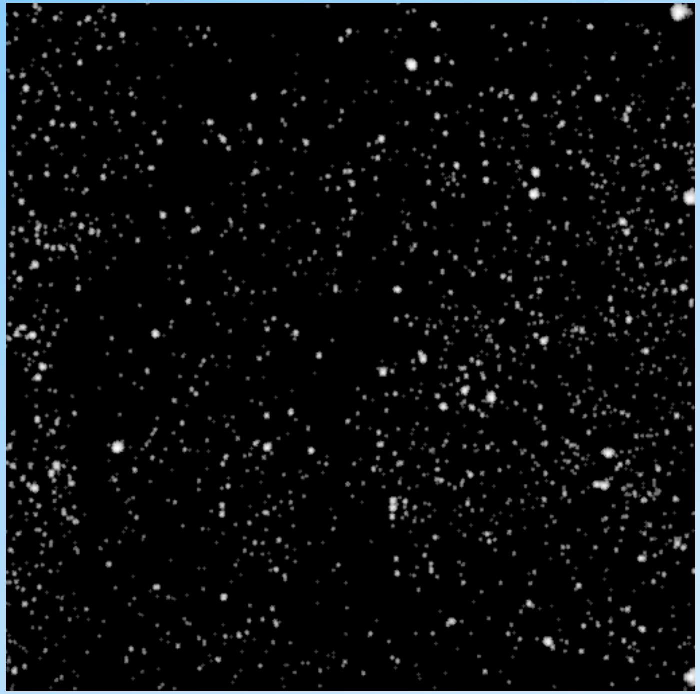
Paoletti,Fiorito,Sreenivasan,and Lathrop, J.Phys. Soc. Jpn. 77,111007(2008)

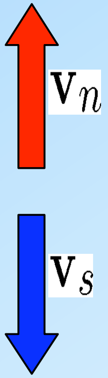
Upward particles



Downward particles







The broken line shows

$$v_n = \frac{q}{\rho S T} \qquad v_s = -\frac{\rho_n}{\rho_s} v_n$$

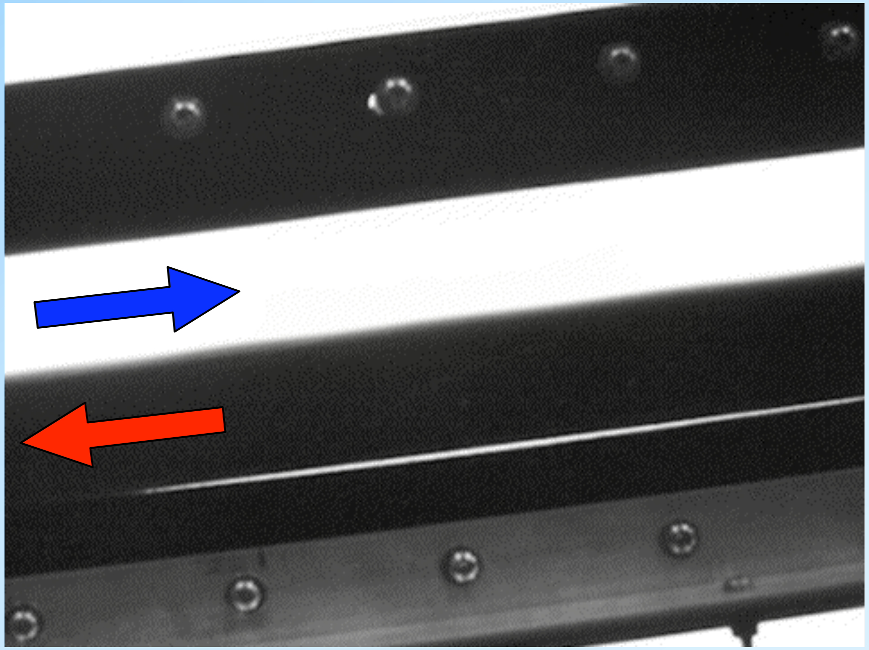
The downward particles should be related with the velocity of vortices!

3.2 Quantum Kelvin-Helmholtz instability in two-component Bose-Einstien condensates

Hiromitsu Takeuchi, Naoya Suzuki, Kenichi Kasamatsu, Hiroki Saito, MT,
Phys. Rev. B (in press): arXiv.0909.2144

KHI: Hydrodynamic instability of shear flows

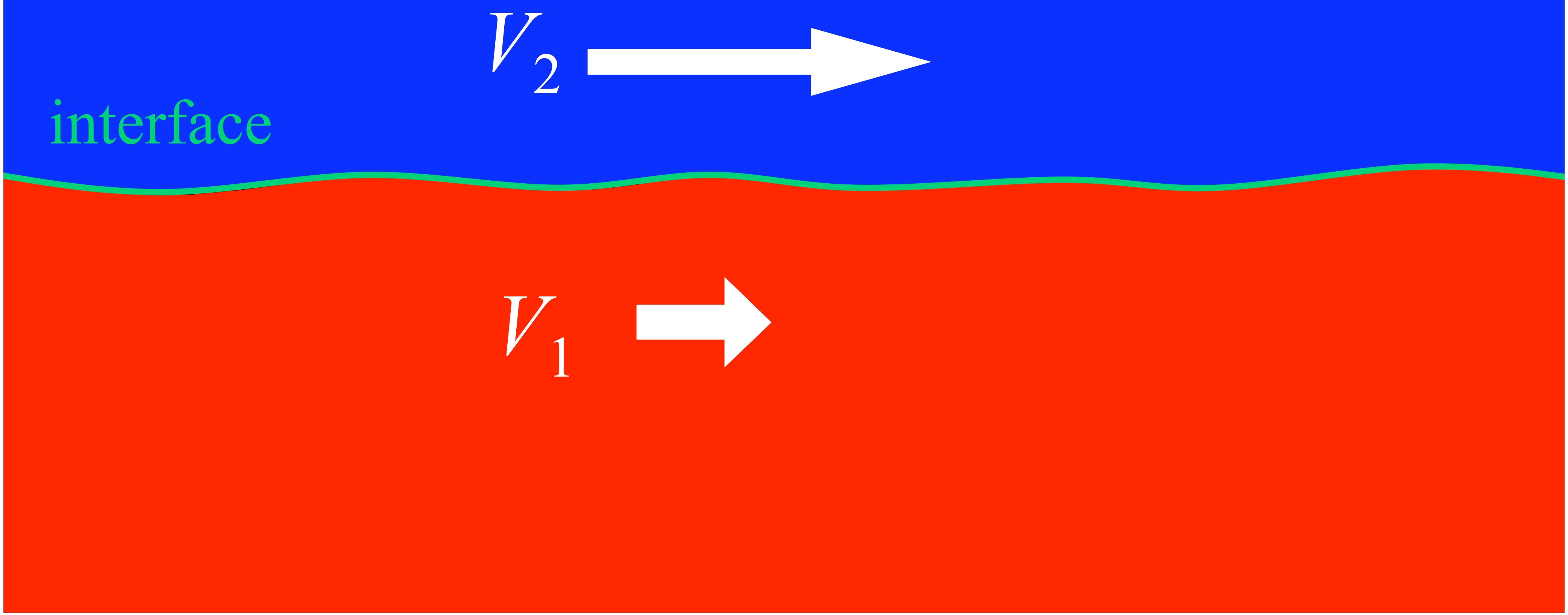
One of the most fundamental instability in classical fluid dynamics



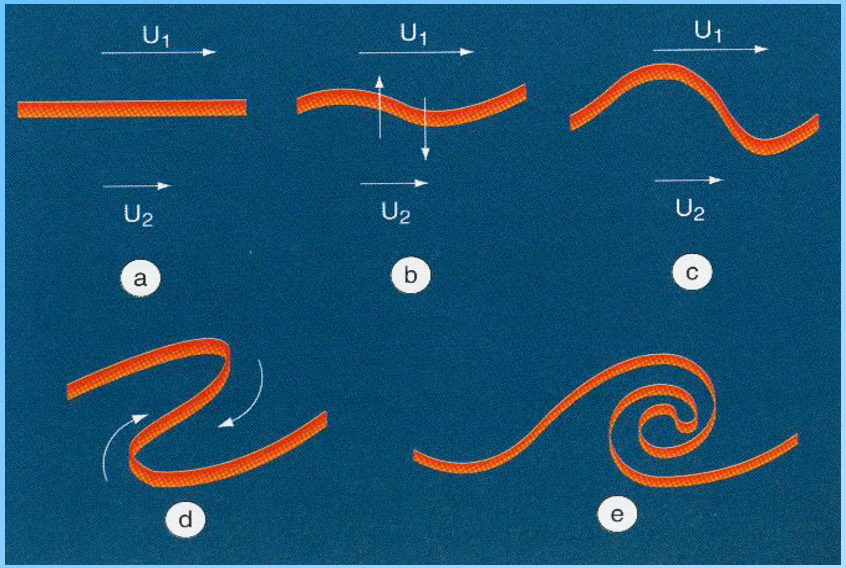
We study the KHI in two-component atomic Bose-Einstein condensates(BECs).

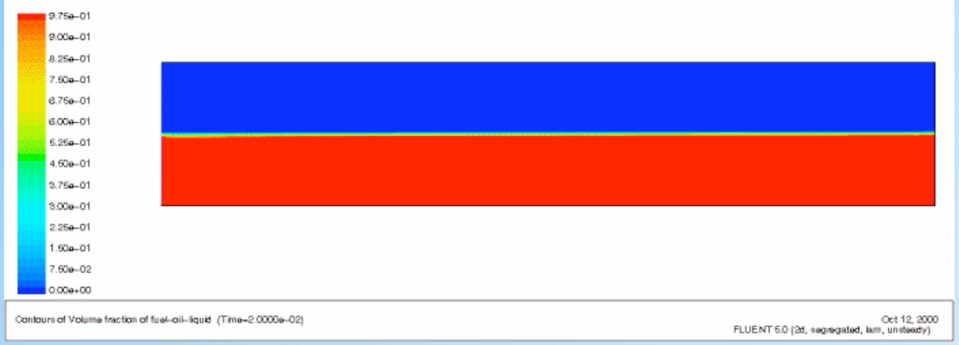
Classical KHI

When the relative velocity $V_d=|V_1-V_2|$ is sufficiently large, the vortex sheet becomes dynamically unstable and the interface modes with complex frequencies are amplified.



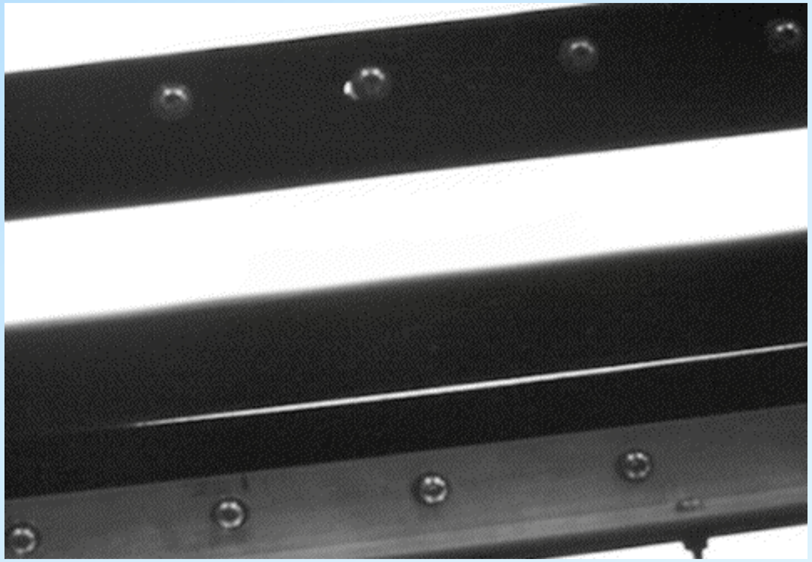
KHI in nature





Contours of Volume fraction of fuel-oil-liquid (Time=2.0000e-02)

Oct 12, 2000
FLUENT 5.0 (2d, segregated, lam, unsteady)

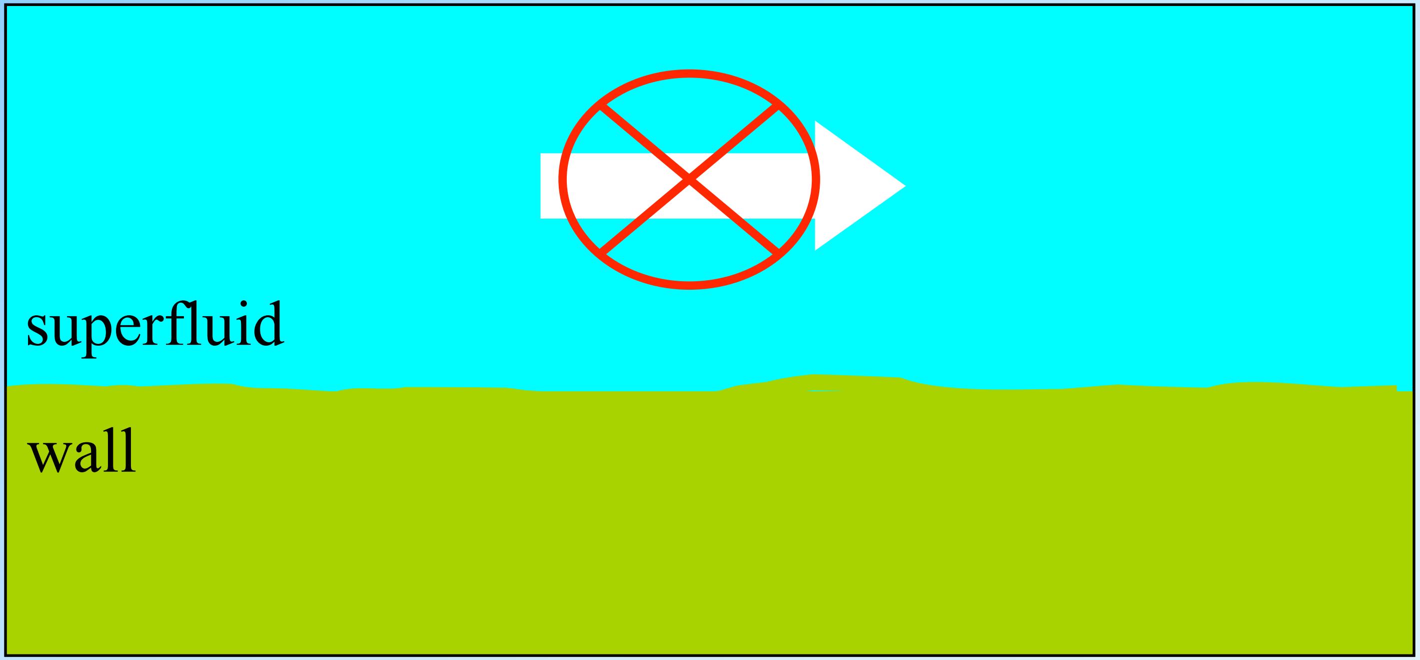


http://hmf.enseeiht.fr/travaux/CD0001/travaux/optmfn/hi/01pa/hyb72/kh/kh_theo.htm

Two important quantum effects in superfluid

1. Superfluidity

Superfluid can flow relative to a wall even in thermal equilibrium like an inviscid fluid.

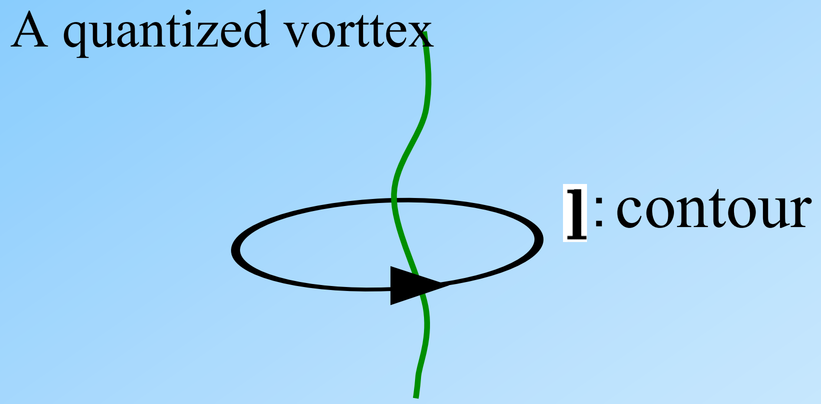


Superfluidity is broken when the relative velocity exceed a critical velocity.

Two important quantum effects in superfluid

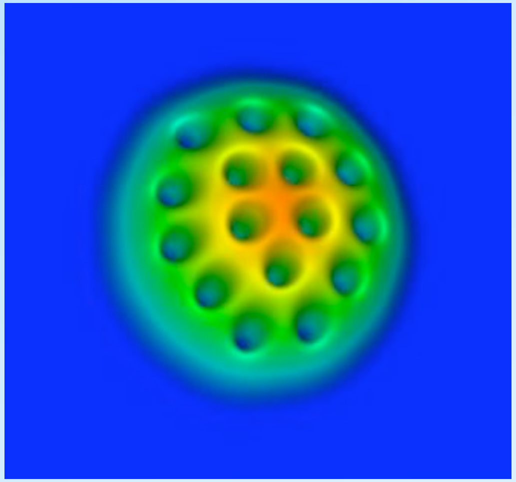
2. Quantized vortex

Vortices appear as topological defects. The circulation around vortices are quantized.



$$\boxed{\Gamma = \oint \mathbf{v} \cdot d\mathbf{l} = \frac{\hbar}{m} \oint (\nabla\theta) \cdot d\mathbf{l} = \frac{h}{m} n}$$

n: interger



MT, K. Kasamatsu, M. Ueda, PRA65, 023603(2002)

Quantum effect on the KHI

stationary shear flow

linear stability **+superfluid stability**

interface wave

nonlinear dynamics **→quantized vortices**

different states or turbulence

Quantum effects play important roles in the quantum KHI.

Two-component BEC

two order parameters (macroscopic wave functions) Ψ_1 Ψ_2

Gross-Pitaevskii(GP) equation

$$i\hbar\partial_t\Psi_1 = \left(-\frac{\hbar^2}{2m_1}\nabla^2 + U_1 + g_{11}|\Psi_1|^2 + g_{12}|\Psi_2|^2\right)\Psi_1$$

$$i\hbar\partial_t\Psi_2 = \left(-\frac{\hbar^2}{2m_2}\nabla^2 + U_2 + g_{12}|\Psi_1|^2 + g_{22}|\Psi_2|^2\right)\Psi_2$$

$g = g_{11} = g_{22}$ $\quad m = m_1 = m_2$

$$\Psi_j(t,\mathbf{r}) = \underbrace{\sqrt{n_j(t,\mathbf{r})}}_{\text{particle density}}\, e^{i\underbrace{\Theta_j(t,\mathbf{r})}_{\text{phase}}}$$

$$\mathbf{v}_j = \frac{\hbar}{m_j}\nabla\Theta_j$$

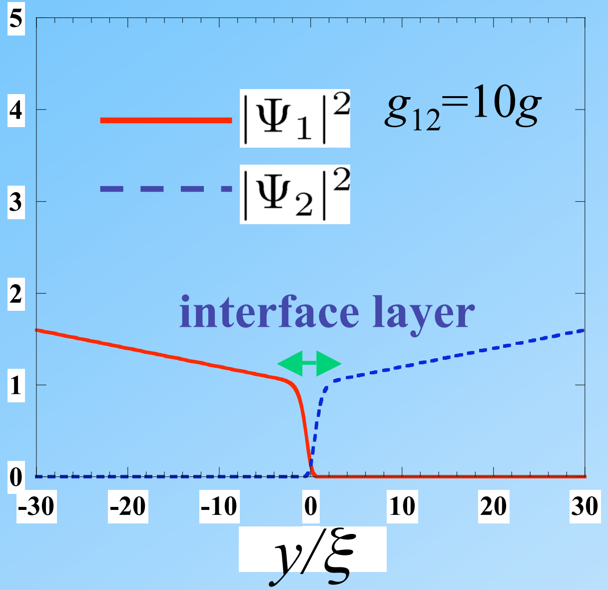
superfluid velocity of component j

Phase-separated two-component BEC

$\Psi_1 \Leftrightarrow \Psi_2$

strong repulsive interaction

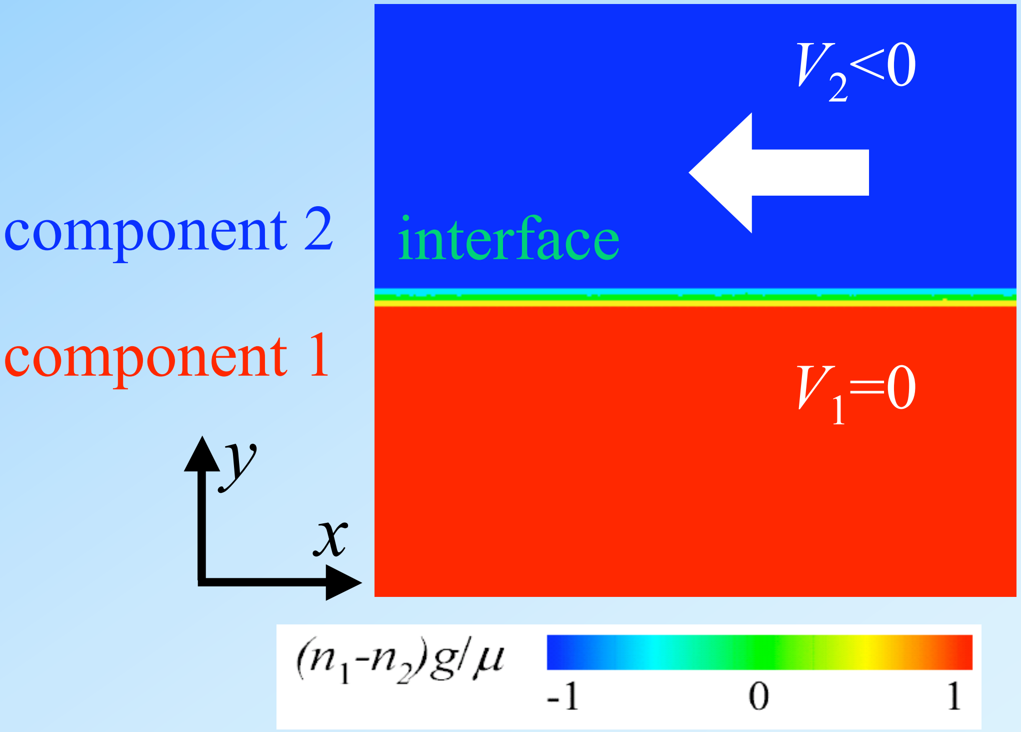
condition for phase separation : $g_{12}>g$



$U_j(y)=f_j y \qquad f_1=-f_2>0$

$\mu=\mu_1-mV_1^2/2=\mu_2-mV_2^2/2>0$

$\xi=\sqrt{\hbar^2/(m\mu)}$



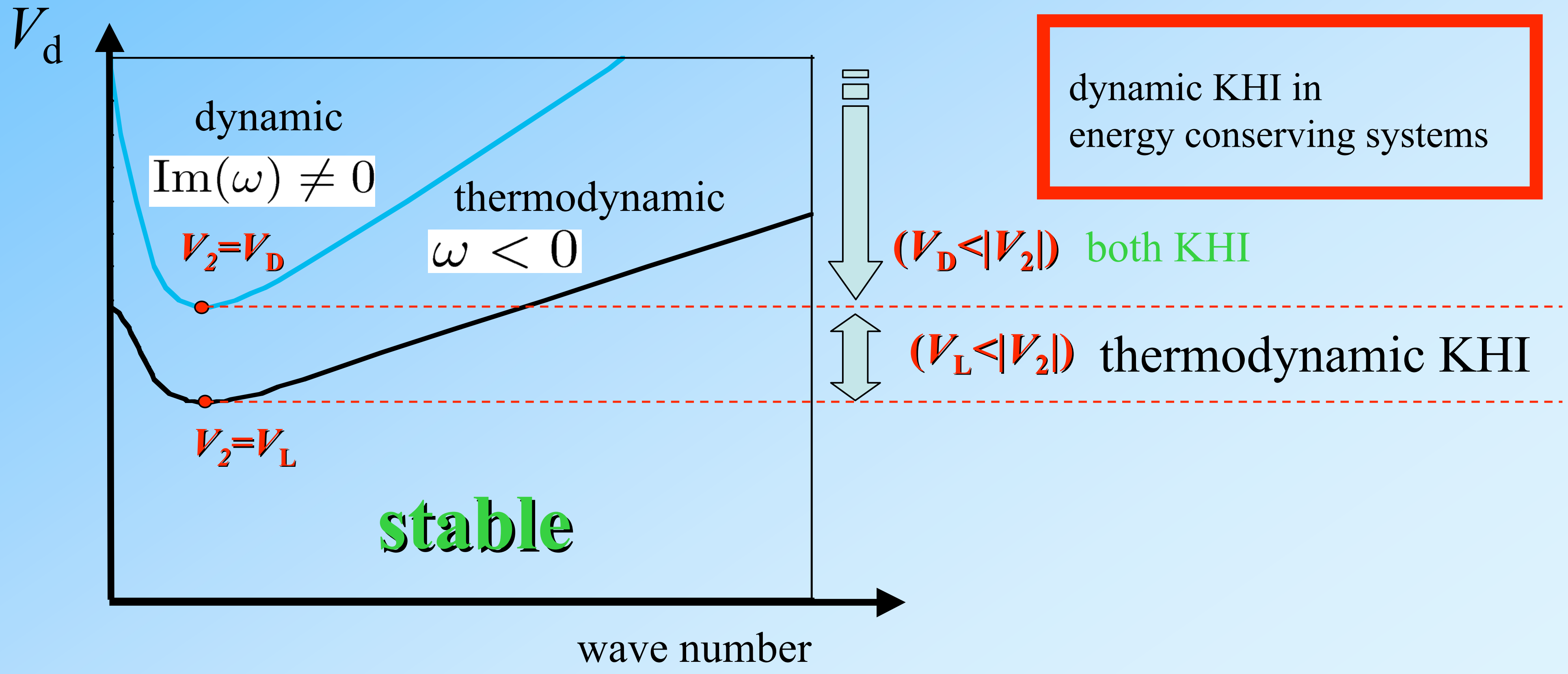
Phase diagram of quantum KHI

-Analysis by the Bogoliubov-de Gennes equations-

dynamic instability $\mathrm{Im}(\omega) \neq 0$ → dynamic KHI（analogue of the classical KHI）

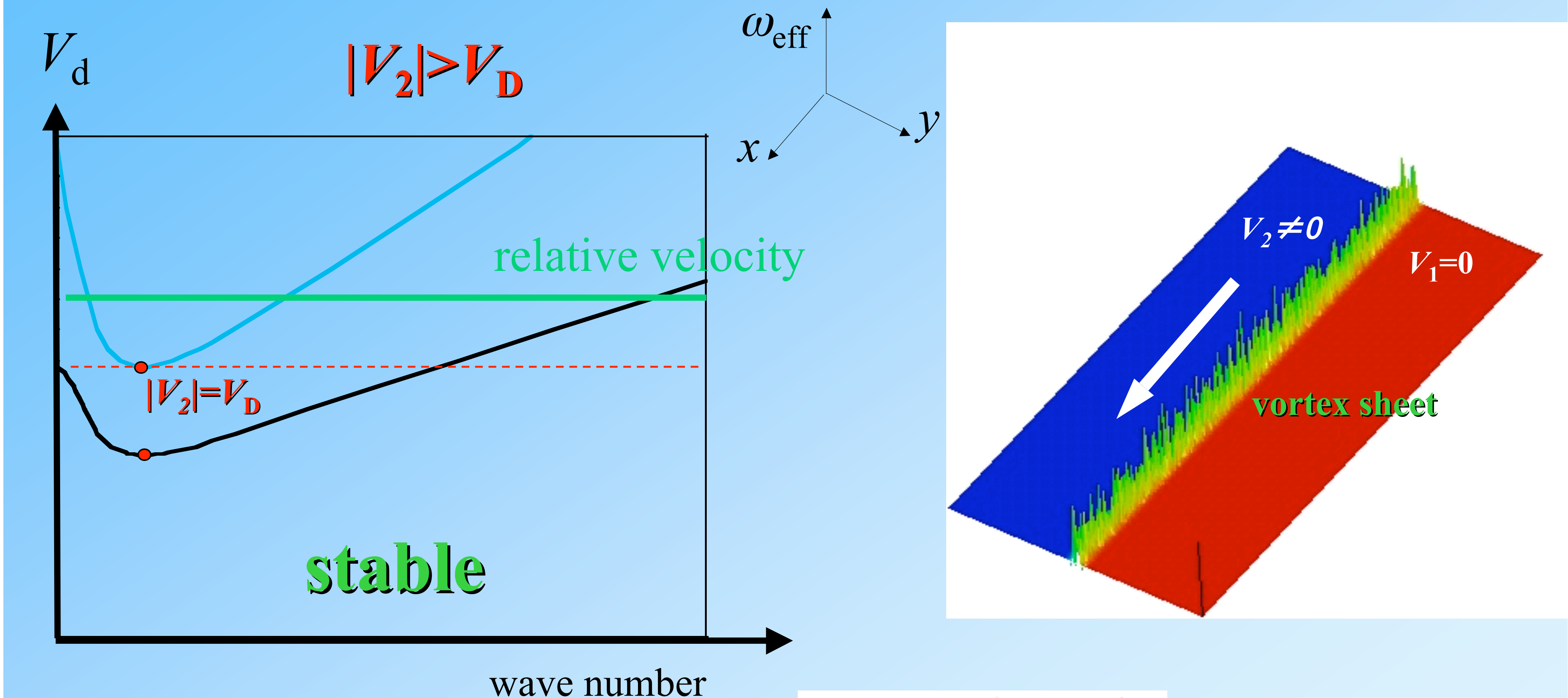
superflow instability $\omega < 0$ → thermodynamic KHI（unique to quantum KHI）

ω: frequency of ripplon



dynamic KHI in
energy conserving systems

Dynamic KHI in energy conserving system

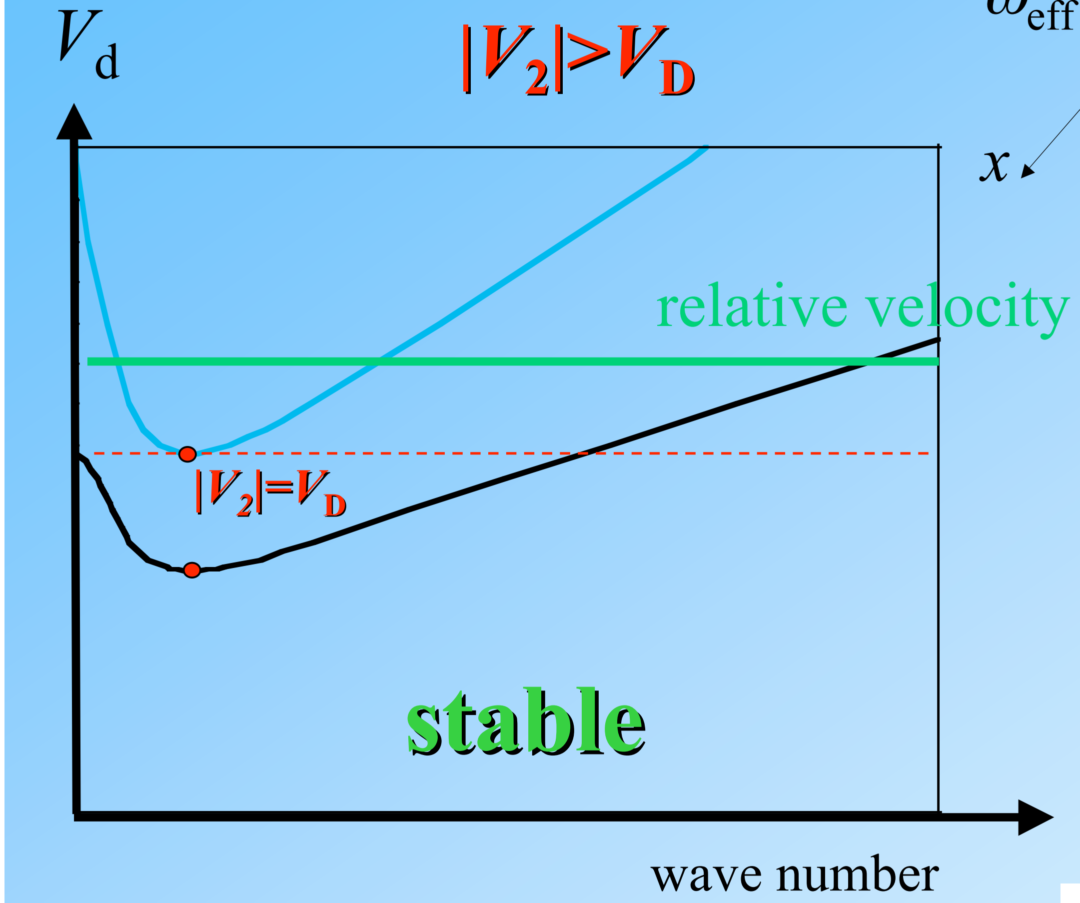


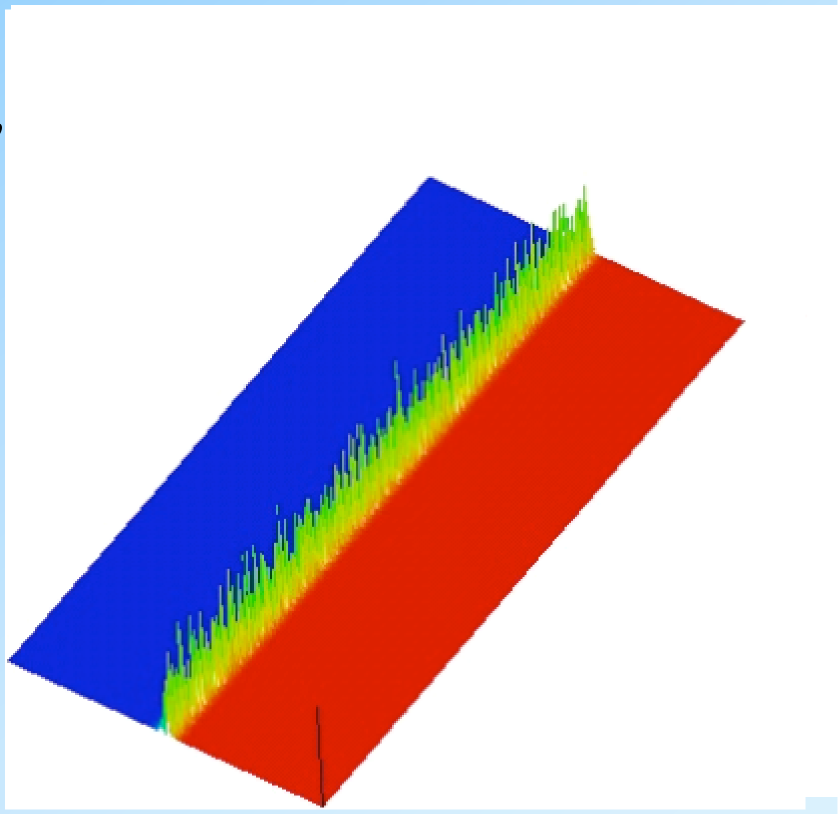
$$\omega_\mathrm{eff} = (\nabla \times \mathbf{v}_\mathrm{eff})_z$$

$$\boldsymbol{v}_\mathrm{eff} = \frac{\boldsymbol{j}_1 + \boldsymbol{j}_2}{n_1 + n_2} \qquad \boldsymbol{j}_i = n_i \boldsymbol{v}_i$$

effective super-current velocity

Dynamic KHI in energy conserving system





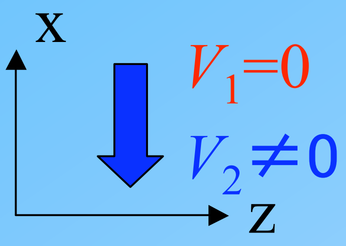
$$\omega_{\rm eff} = (\nabla \times \mathbf{v}_{\rm eff})_z$$

$$\boldsymbol{v}_{\rm eff} = \frac{\boldsymbol{j}_1 + \boldsymbol{j}_2}{n_1 + n_2}$$

$$\boldsymbol{j}_i = n_i \boldsymbol{v}_i$$

effective super-current velocity

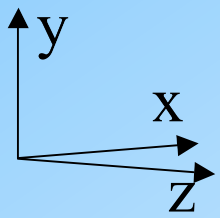
Dynamic KHI in 3D system

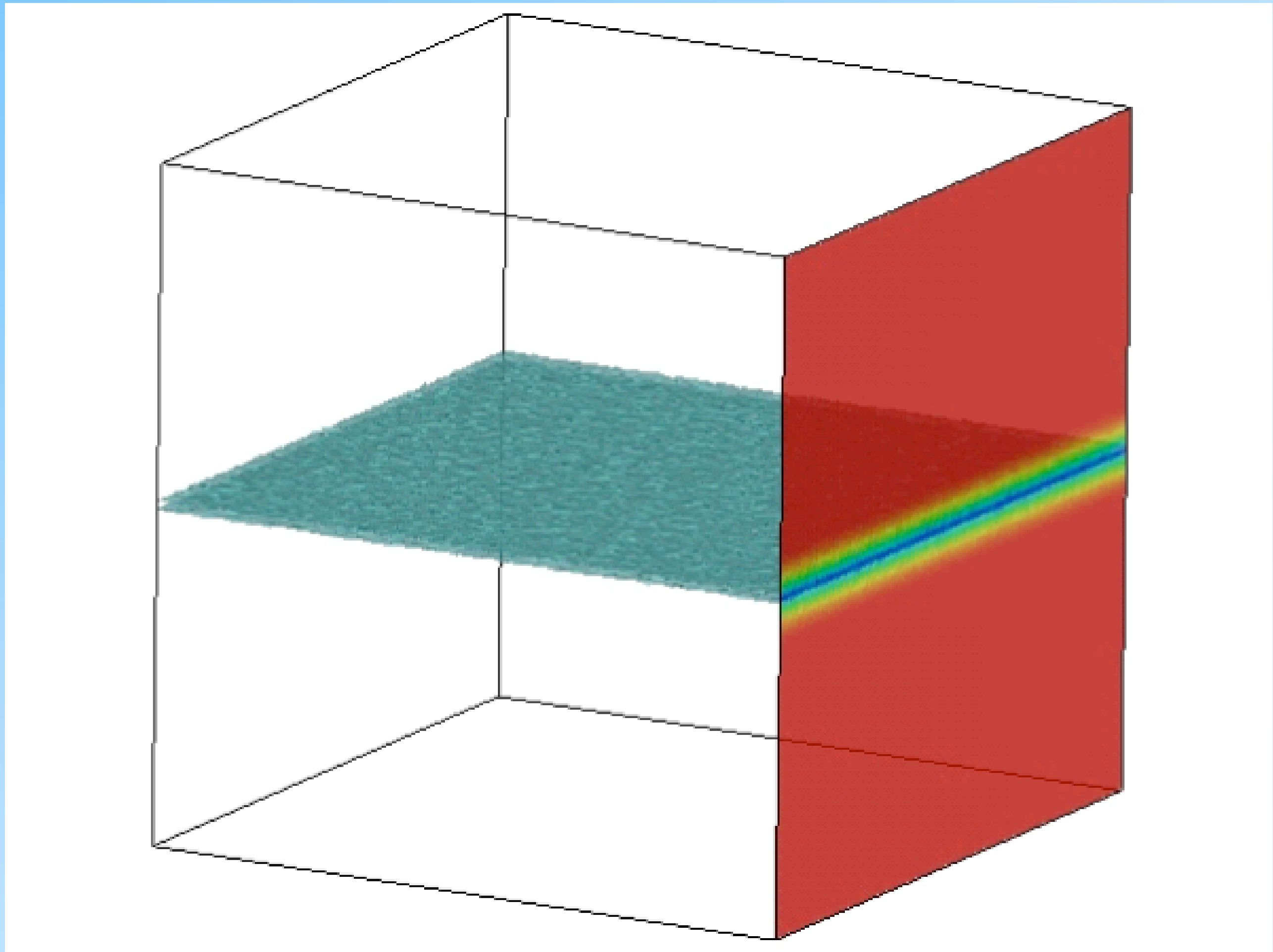


$V_1=0$

$V_2\neq 0$

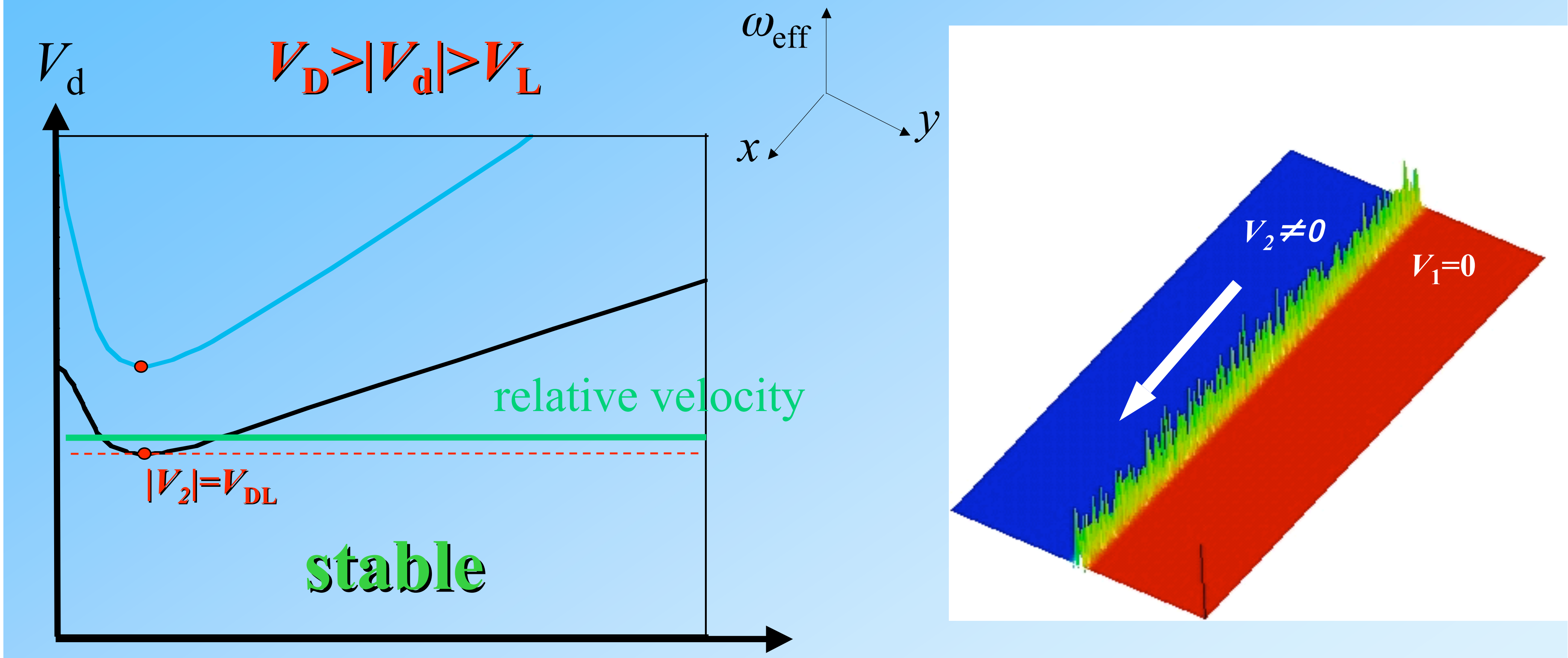
$|V_2|>V_D$





Kelvin waves cause more complicated dynamics towards quantum turbulence.

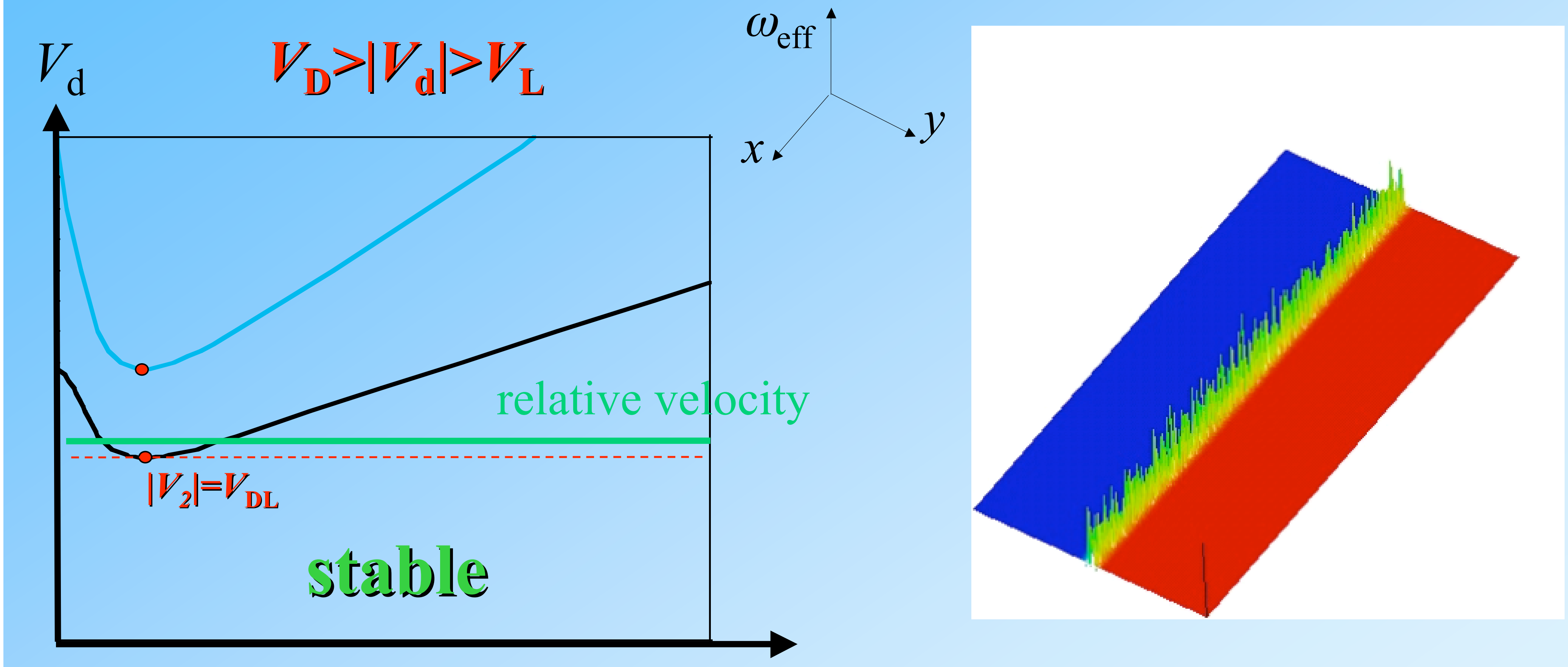
Thermodynamic KHI in dissipative system



GP model with dissipation

K. Kasamatsu, M. Tsubota M. Ueda, Phys. Rev. A **67**, 033610 (2003).

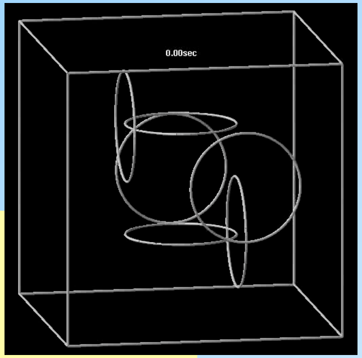
Thermodynamic KHI in dissipative system



GP model with dissipation

K. Kasamatsu, M. Tsubota M. Ueda, Phys. Rev. A **67**, 033610 (2003).

Summary



1. Why is QT (quantum turbulence) so important?
2. Outputs of our group through this five-years project
3. Very new results

3.1 Steady state of counterflow quantum turbulence: Vortex filament simulation with the full Biot-Savart law
H. Adachi, S. Fujiyama, MT, Phys. Rev. B (in press) (***Editors suggestion***): arXiv:0912.4822

3.2 Quantum Kelvin-Helmholtz instability in two-component Bose-Einstien condensates
H. Takeuchi, N. Suzuki, K. Kasamatsu, H. Saito, MT,
Phys. Rev. B (in press): arXiv:0909.2144