# Boundary and Impurity Effects on Fourth Sound Propagation in Superfluid $^3$He

Seiji Higashitani

Graduate School of Integrated Arts and Sciences, Hiroshima University

Collaborators

Hiroshima Univ. K. Nagai
Y. Nagato
Osaka City Univ. O. Ishikawa
K. Obara
C. Kato

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# Normal liquid $^3$He between parallel plates

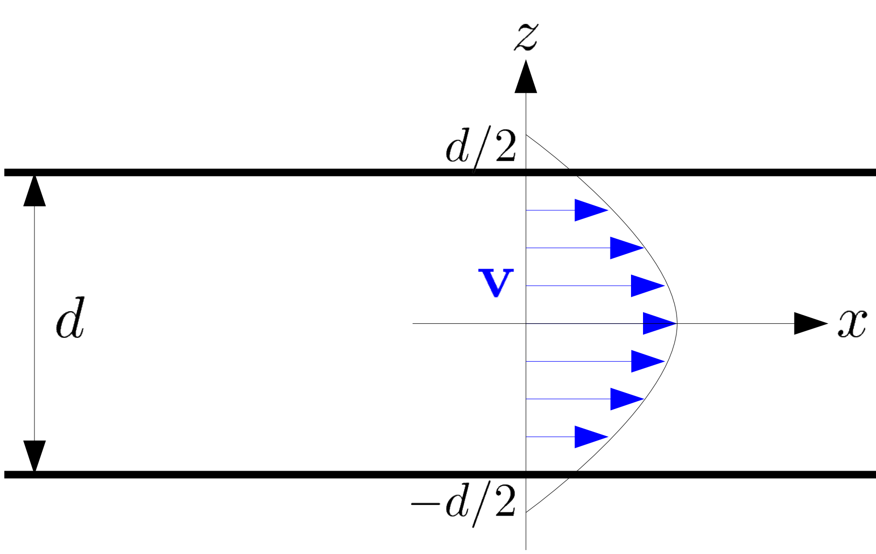
Navier-Stokes equation

$$\rho \partial_t \mathbf{v} = -\nabla P + \eta \nabla^2 \mathbf{v}$$

Boundary conditions

$$v_x(\pm d/2) = \mp \zeta v'_x(\pm d/2) \quad (\zeta : \text{slip length})$$

Hagen-Poiseuille (HP) flow ( $\omega$ = 0) $\quad v_x(z) = \left(z^2 - (d/2)^2 - \zeta d\right) \dfrac{\partial_z P}{2\eta}$



# Normal liquid $^3$He in aerogel

$$\rho \partial_t \mathbf{v} = -\nabla P + \eta \nabla^2 \mathbf{v} - \frac{1}{\tau_f} \rho \mathbf{v}$$

Friction with aerogel

Viscous penetration depth $\delta_v = \dfrac{1}{\mathrm{Im}\sqrt{\dfrac{\rho\,(i\omega - 1/\tau_f)}{\eta}}}$

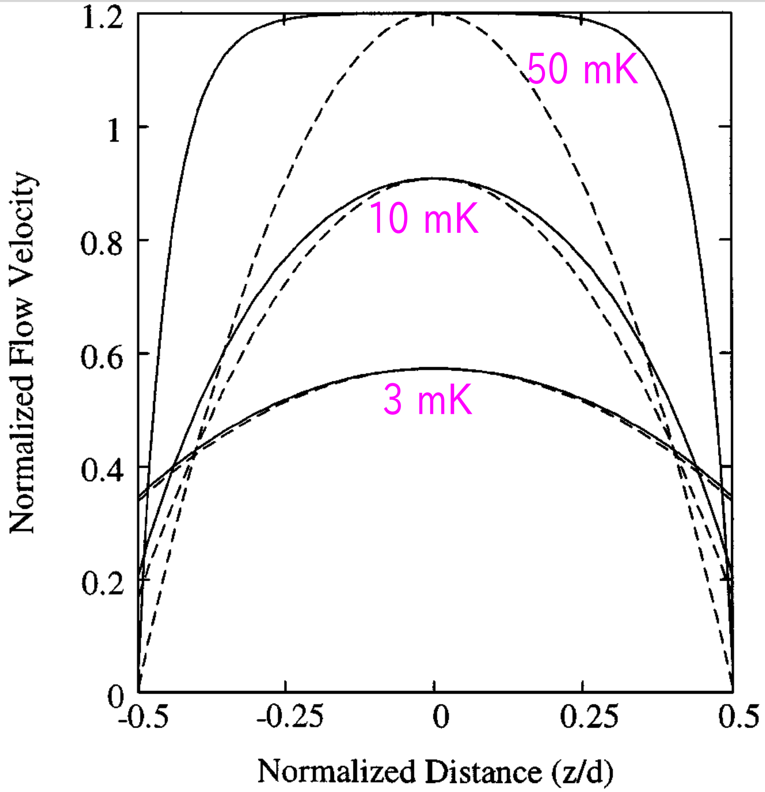
$$\delta_v = \begin{cases} \sqrt{\dfrac{2\eta}{\rho\omega}} & (\omega \gg 1/\tau_f) \\ \sqrt{\dfrac{\eta\tau_f}{\rho}} & (\omega \ll 1/\tau_f) \end{cases}$$

$(\omega \gg 1/\tau_f)$ ⇒ pure liquid $^3$He

$(\omega \ll 1/\tau_f)$ ⇒ Even in the low frequency limit, $\delta_v$ takes a finite value.

$$\eta \propto \tau_\eta = \frac{1}{1/\tau_\eta^{\mathrm{pure}} + 1/\tau_{\mathrm{imp}}} \sim \frac{1}{T^2 + \mathrm{const}}$$

# Flow profile between a parallel plates




D. Einzel and J. M. Parpia, PRL **81**, 3896 (1998)

FIG. 2. The normalized profiles $g_x(z)/nF_{mx}\bar{\tau}^i(1\text{ mK})$ for $^3$He at 30 bars with 99.9% open aerogel at 3 mK (lower), 10 mK (middle), and 50 mK (upper curves). Full lines represent the general result including slip, and dashed lines are the scaled profiles $[g_x^0(z)/g_x^0(0)]g_x(0)$ in the absence of aerogel, with $g_x^0(z) = \lim_{\ell^*\to\infty} g_x(z) = nF_{mx}\tau_{m1}^*(d^2/8\ell^{*2})[1 + 4(\zeta_0/d - z^2/d^2)]$. The dashed lines are normalized to the calculated flow in the center of the channel and illustrate the difference between Poiseuille flow and the flow with aerogel present.

$$d = 2\ \mu\text{m}$$

$$\delta_v = \sqrt{\eta\tau_f/\rho} \propto \sqrt{1/(T^2+\text{const})}$$

$$(\omega = 0)$$

## Cross sectional average of the mass current

$$g_x(z) = \rho v_x(z) = mnv_x(z)$$

$$\langle g_x\rangle = \frac{1}{d}\int_{-d/2}^{d/2} dz\, g_x(z)$$

$$= n\tau^{\text{eff}}F^{\text{ext}} \quad (F^{\text{ext}} = -\partial_x P/n)$$

## Effective flow relaxation time (flow conductance)

$$\tau^{\text{eff}} = \begin{cases} \dfrac{\rho d^2}{12\eta}\left(1+\dfrac{6\zeta}{d}\right) & \delta_v \gg d \text{ (HP law)} \\ \\ \tau_f \sim \tau_{\text{imp}} & \delta_v \ll d \text{ (Drude law)} \end{cases}$$

# Fourth sound propagation in superfluid $^3$He

Two-fluid model

$$\partial_t \rho + \nabla \cdot (\rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s) = 0$$

$$\partial_t \mathbf{v}_s = -\frac{1}{\rho} \nabla P$$

$$\rho_n \partial_t \mathbf{v}_n = -\frac{\rho_n}{\rho} \nabla P + \eta \nabla^2 \mathbf{v}_n - \frac{1}{\tau_f} \rho_n \mathbf{v}_n$$

Dispersion relation

$$\omega^2 = c_4^2 q^2 + \rho_n \omega q \frac{\langle v_{nx} \rangle}{\langle \delta \rho \rangle}$$

$\langle \cdots \rangle$ : cross sectional average

$c_4 = \sqrt{\frac{\rho_s}{\rho}}\, c_1$ (4th sound velocity)

# Dispersion relation

- $\delta_v \gg d$ (HP)

$$\omega^2 = c_4^2 q^2 \left[1 - i\frac{\rho_n}{\rho_s}\frac{\rho_n \omega d^2}{12\eta}\left(1 + \frac{6\zeta}{d}\right)\right]$$

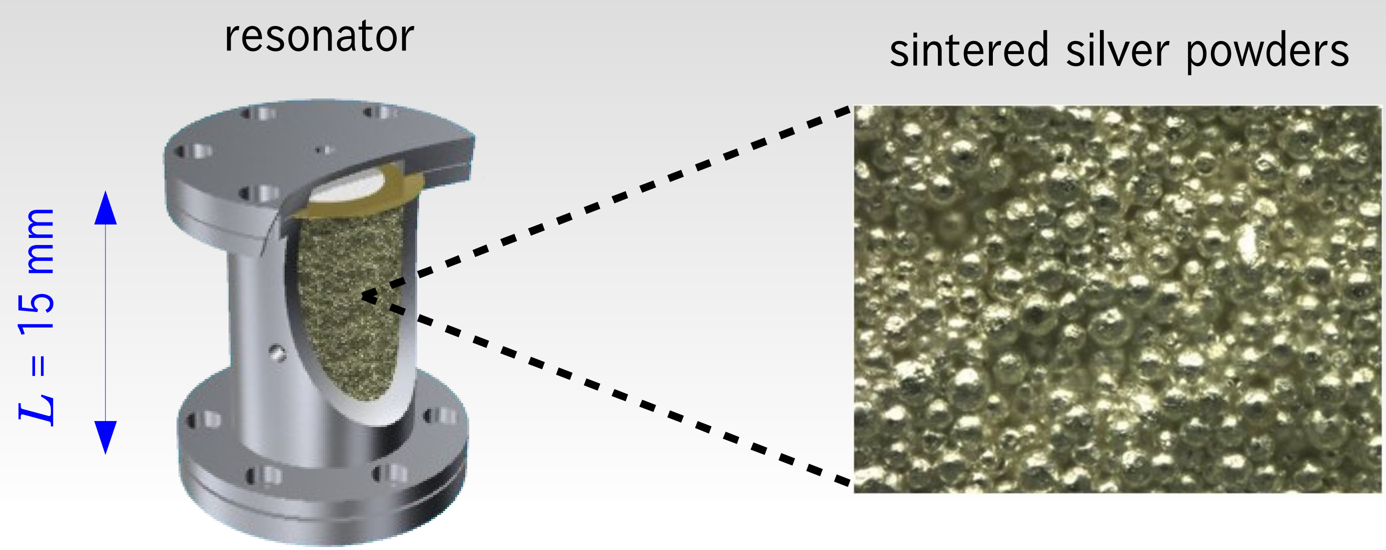
- $\delta_v \ll d$ (Drude)

$$\omega^2 = c_4^2 q^2 \left(1 - i\frac{\rho_n}{\rho_s}\omega\tau_f\right)$$

viscous penetration depth: $\delta_v = \dfrac{1}{\mathrm{Im}\sqrt{\dfrac{\rho_n\left(i\omega - 1/\tau_f\right)}{\eta}}}$

fourth sound velocity: $c_4 = \sqrt{\dfrac{\rho_s}{\rho}}\, c_1$

# Fourth sound resonance experiment



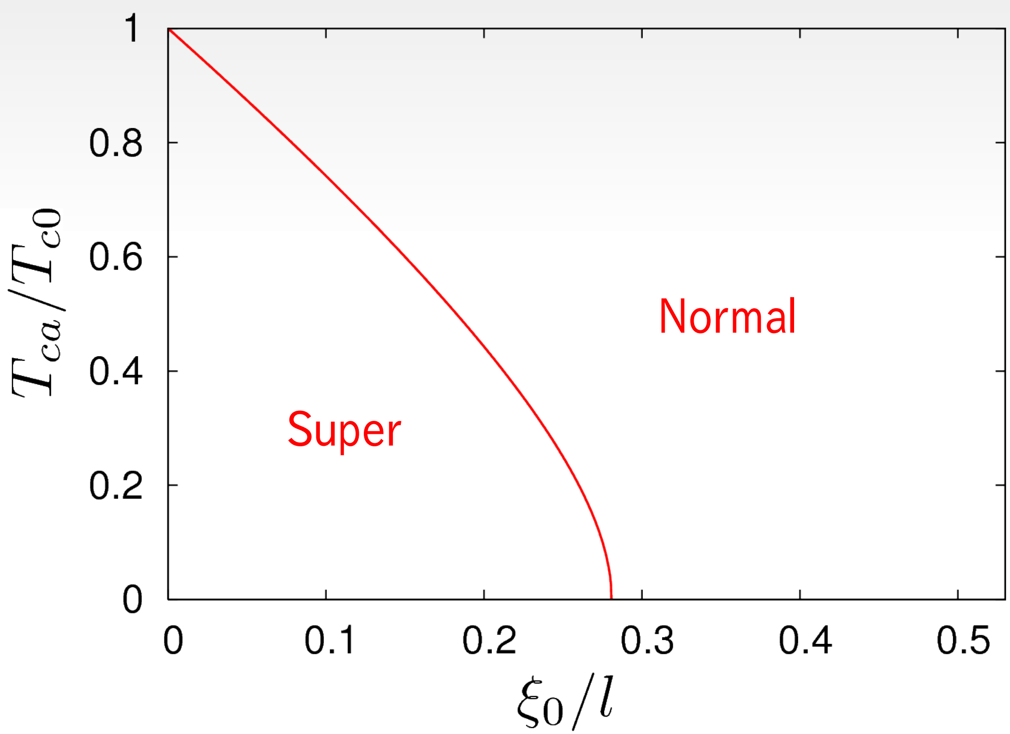

- Aerogel (99 % porosity) is embedded in pores formed by sintered silver powders.
- The pore size is $\sim 10\ \mu$m

$$\omega^2 = c_4^2 q^2 (1 - iQ^{-1}) \qquad c_4 = \sqrt{\frac{\rho_s}{\rho}}\, c_1 \qquad q = \frac{\pi}{L} n_{\mathrm{mode}}$$

$Q^{-1}$ : energy loss

# $T_c$ reduction

$$\ln \frac{T_{ca}}{T_{c0}} = -\sum_{n=0}^{\infty} \left( \frac{1}{n+\frac{1}{2}} - \frac{1}{n+\frac{1}{2}+\frac{1}{2}\frac{\xi_0}{l}\frac{T_{c0}}{T_{ca}}} \right)$$




$$\xi_0 = v_F/2\pi T_{c0} = 17 \text{ nm at 29 bar}$$

99 % aerogel 29 bar

$$T_{c0} = 2.42 \text{ mK}$$

$$T_{ca} = 2.27 \text{ mK}$$

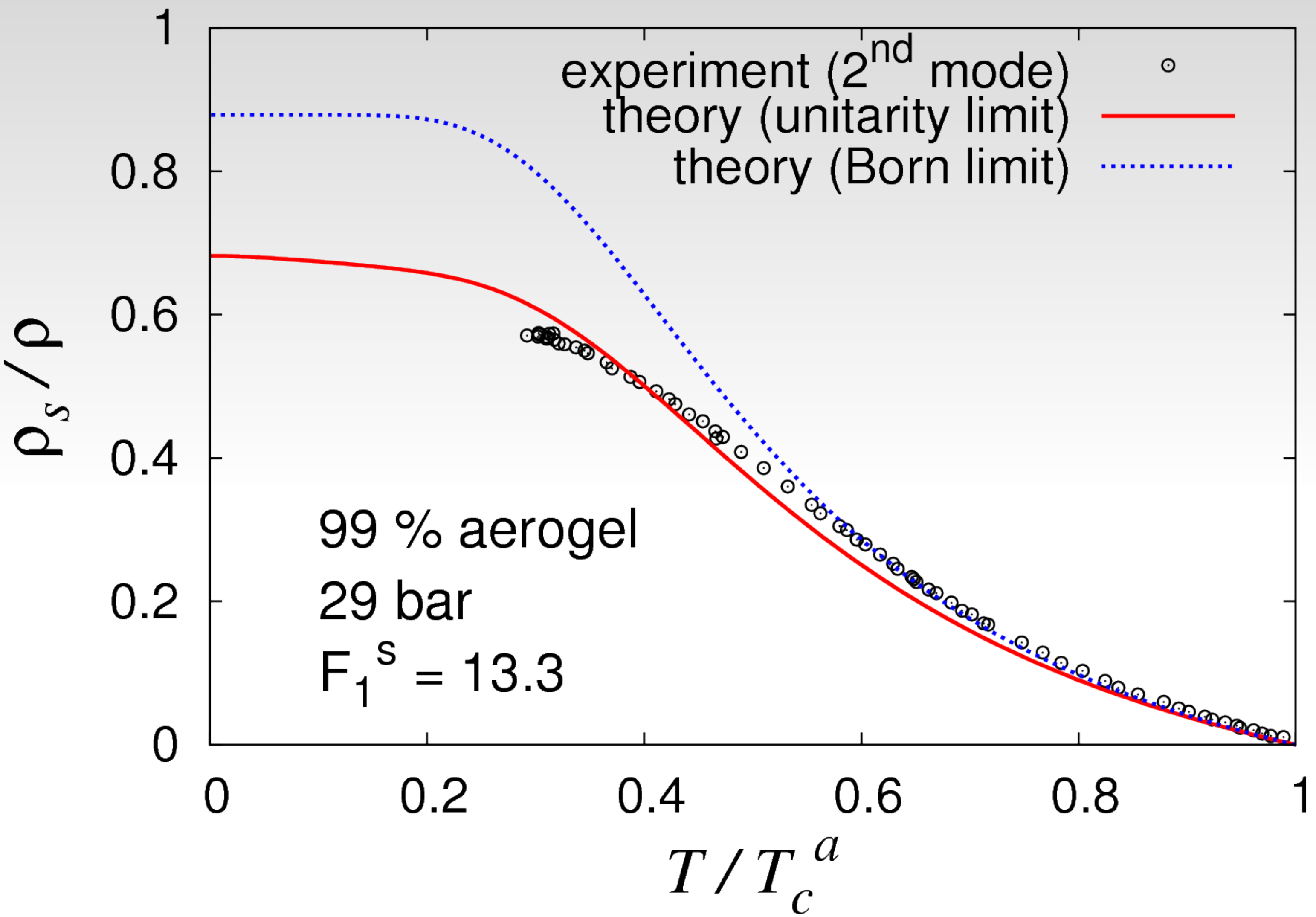
$$T_{ca}/T_{c0} = 0.94$$

$$l = v_F \tau_{\text{imp}} \sim 700 \text{ nm}$$

$$(\tau_{\text{imp}} \sim 20 \text{ ns}, \ v_F = 34 \text{ m/s})$$

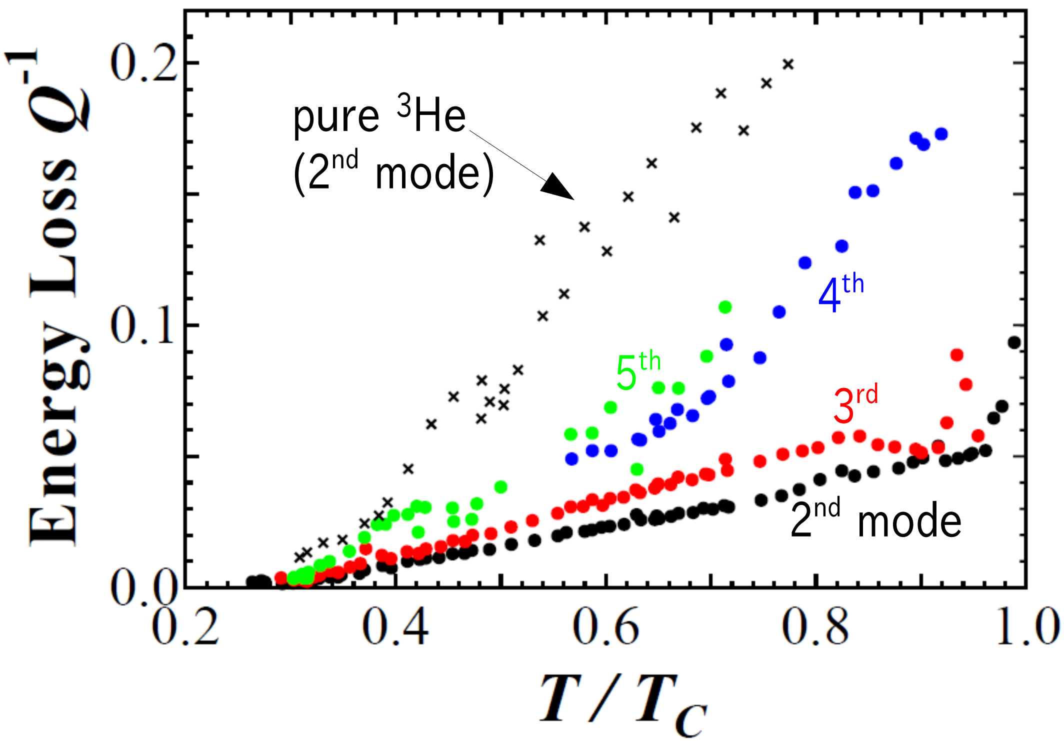
$$l \sim 150 \text{ nm (98 \% aerogel)}$$

# Superfluid density




Temperature dependence of $\rho_s / \rho$ in superfluid $^3$He-B

# Energy loss of the fourth sound




- $Q^{-1}$ is much reduced by the impurity effect.
- $Q^{-1}_{aerogel}$ decreases to zero in the low temperature limit, as in the case of $Q^{-1}_{pure}$.

$$\omega^2 = c_4^2 q^2 (1 - iQ^{-1})$$

$$Q^{-1} = \begin{cases} \dfrac{\rho_n}{\rho_s} \dfrac{\rho_n \omega d^2}{12\eta} \left(1 + \dfrac{6\zeta}{d}\right) & \text{(HP)} \\ \dfrac{\rho_n}{\rho_s} \omega \tau_f & \text{(Drude)} \end{cases}$$

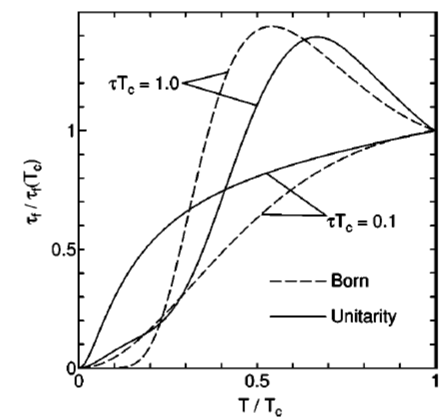


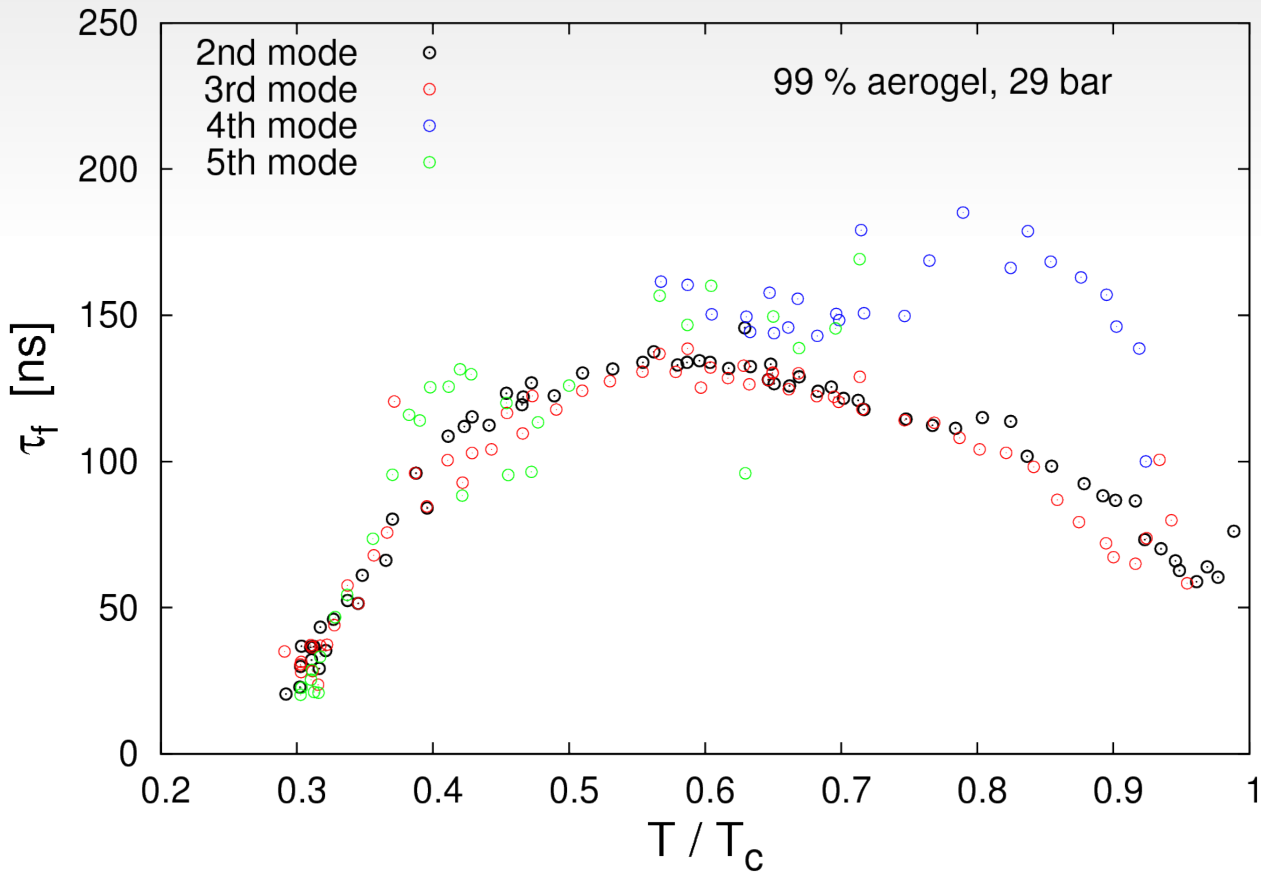

FIG. 2. The frictional relaxation time $\tau_f$ normalized at $T = T_c$ as a function of reduced temperature $T/T_c$. The values of parameters $\tau T_c$ and $F_1^s$ are taken to be $\tau T_c = 0.1, 1.0$ and $F_1^s = 10.1$ (corresponding to a pressure of 16 bar).[34]

Higashitani *et al.*, PRB **71**, 134508 (2005)

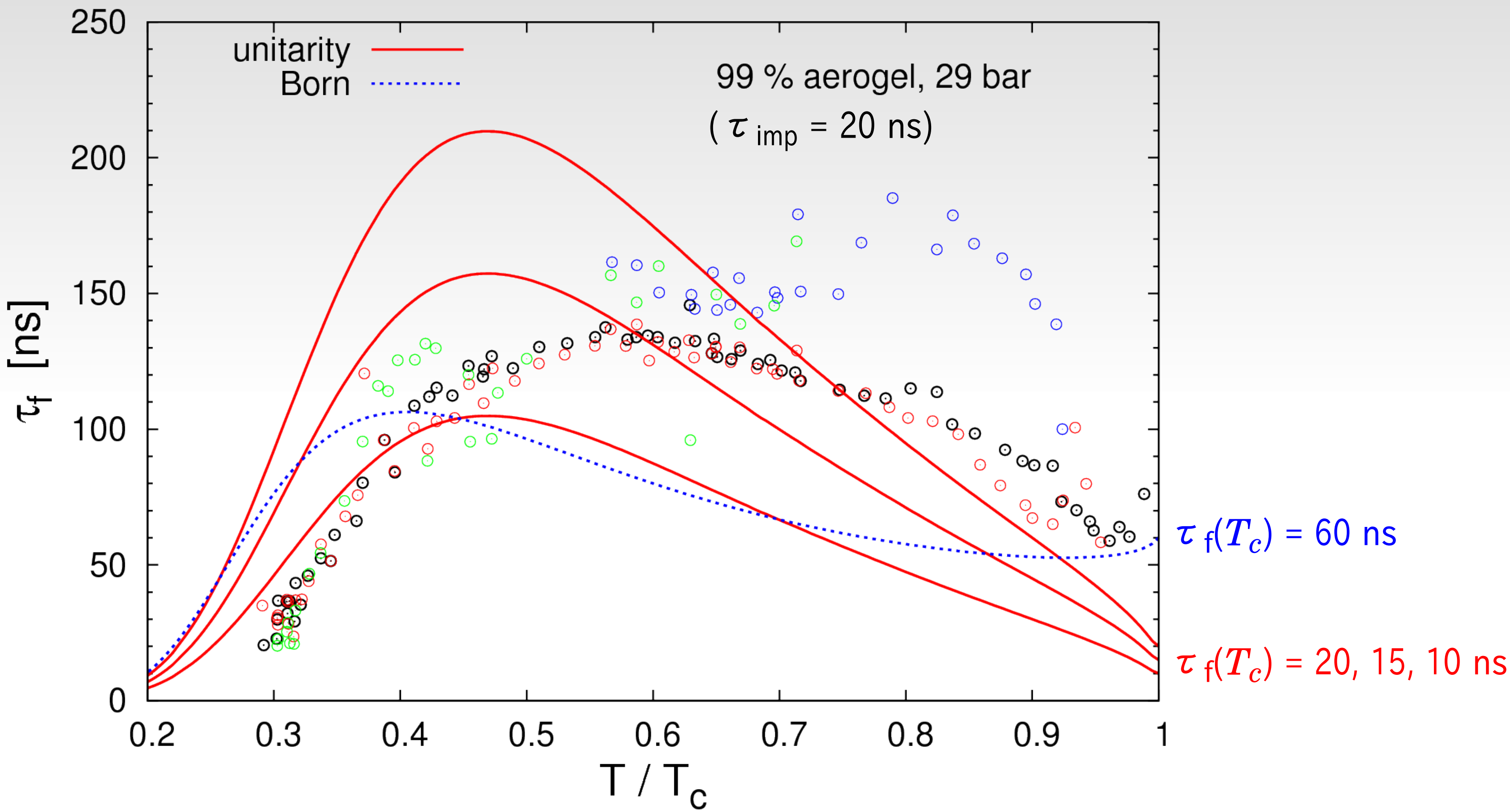
# Frictional relaxation time

$$Q^{-1} = \frac{\rho_n}{\rho_s}\omega\tau_f \quad \Rightarrow \quad \tau_f = \frac{Q^{-1}}{\omega}\frac{\rho_s}{\rho_n}$$

$$\omega = c_4 q = \sqrt{\frac{\rho_s}{\rho}}\, c_1 q \qquad q = \frac{\pi}{L} n_{\mathrm{mode}} \quad (L = 15\ \mathrm{mm})$$



## Temperature dependence of $\tau_f$ in superfluid $^3$He-B



# The normal-fluid dynamics in aerogel

$$\tau_f(T_c) \sim \tau_{\rm imp} \quad \omega \sim 10 \text{ kHz}$$
$$(\tau_{\rm imp} \sim 20 \text{ nm})$$

$\omega\tau_f \ll 1$

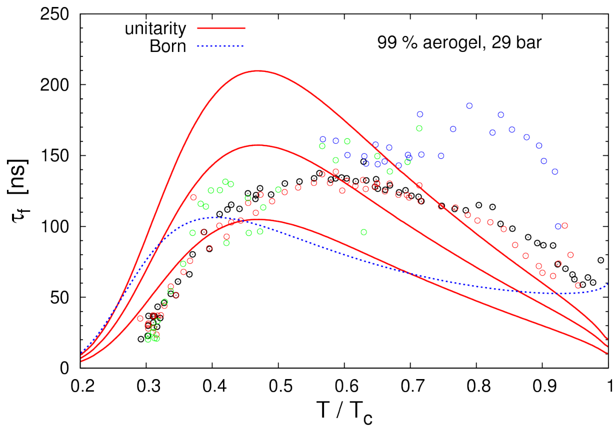
The normal-fluid component is clamped by friction with aerogel.

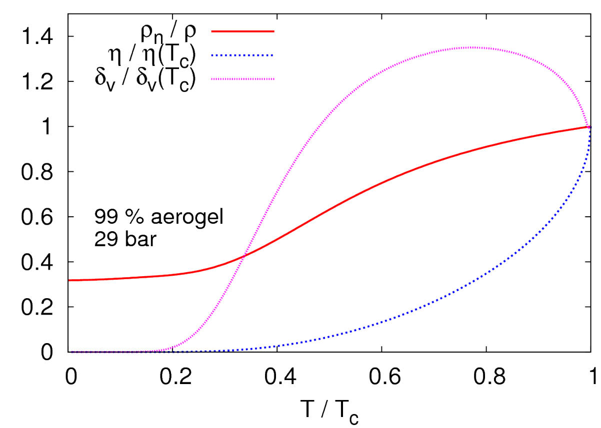
$$\delta_v = \sqrt{\frac{\eta\tau_f}{\rho_n}} \quad \delta_v(T_c) \sim 1\ \mu\text{m}$$
$$d \sim 10\ \mu\text{m}$$

$\delta_v \ll d$

The normal-fluid velocity profile is almost constant across the pore.

(Drude's law)



# Summary

- We have analyzed the fourth sound resonance experiment on superfluid $^3$He in 99 % porosity aerogel.
- The mean free path $l = v_F \tau_{\mathrm{imp}}$ in the 99 % aerogel is ~ 700 nm.
- The normal-fluid dynamics underlying in the fourth sound propagation is governed not by the conventional HP law but by the Drude law.
- The energy loss formula for the Drude law is independent of the pore size *d*. The observation of the *d*-independence of the energy loss $Q^{-1}$ gives another strong evidence of the Drude law.

$$Q^{-1} = \begin{cases} \dfrac{\rho_n}{\rho_s} \dfrac{\rho_n \omega d^2}{12\eta} \left(1 + \dfrac{6\zeta}{d}\right) & \text{(HP)} \\ \\ \dfrac{\rho_n}{\rho_s} \omega \tau_f & \text{(Drude)} \end{cases}$$